

Metod zamene elemenata

Primenjuje se za određivanje reakcija oslonaca i sila u presecima složenih statički određenih nosača kao i statički određenih nosača II vrste.

Proračun uticaja u datom nosaču svodimo na proračun uticaja u nekom drugom jednostavnijem nosaču – *zamenjujućem nosaču*. Ovaj nosač mora biti kinematički stabilan i takav da se reakcije oslonaca i sile u presecima mogu odrediti elementarnim statičkim metodama (metod dekompozicije, čvorova, preseka).

Zamenjujuće nosač dobijamo uklanjanjem određenog broja elemenata na datom nosaču i zamenjujemo ih sa istim brojem novih elemenata.

- Uklonjeni elementi: X_1, X_2, \dots, X_n
- Dodati elementi: Y_1, Y_2, \dots, Y_n

Reakcije dodatih elemenata: (Y)

$$Y_i = Y_{i0} + Y_{i1}X_1 + Y_{i2}X_2 + \dots + Y_{in}X_n = 0$$

Reakcije Y_i ne postoje na stvarnom nosaču pa moraju biti =0

Y_{i0} - reakcija u dodatom elementu i usled spoljašnjeg opterećenja na zamenjujućem nosaču

Y_{in} - reakcija u dodatom elementu i usled $X_n = 1$ na zamenjujućem nosaču

$$Y_{10} + Y_{11}X_1 + Y_{12}X_2 + \dots + Y_{1n}X_n = 0$$

$$Y_{20} + Y_{21}X_1 + Y_{22}X_2 + \dots + Y_{2n}X_n = 0$$

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$$Y_{n0} + Y_{n1}X_1 + Y_{n2}X_2 + \dots + Y_{nn}X_n = 0$$



$$X_1, X_2, \dots, X_n$$

$$\begin{vmatrix} Y_{10} & Y_{11} & Y_{12} & \dots & Y_{1n} \\ Y_{20} & Y_{21} & Y_{22} & \dots & Y_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ Y_{n0} & Y_{n1} & Y_{n2} & \dots & Y_{nn} \end{vmatrix} \neq 0$$

Sistem jednačina ima rešenje i nosač je kinematički stabilan.

Uticaje u nosaču određujemo principom super pozicije:

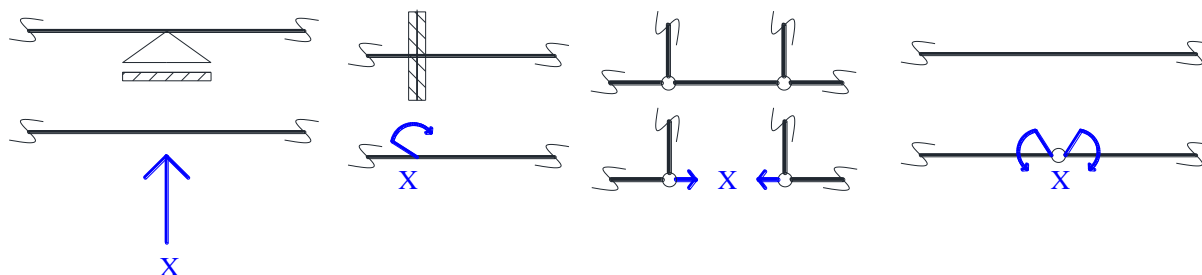
$$Z = Z_0 + Z_1X_1 + Z_2X_2 + \dots + Z_nX_n$$

Z – uticaji u zadatom nosaču (reakcije oslonaca, presečne sile)

Z_0 - uticaj Z na zamenjujućem nosaču usled spoljašnjeg opterećenja

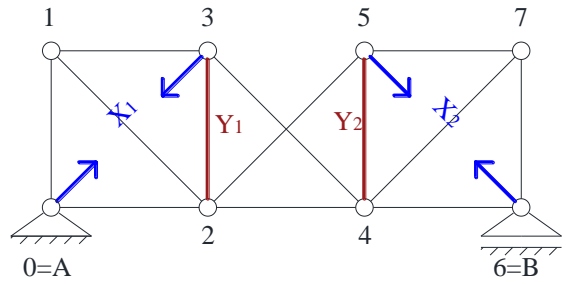
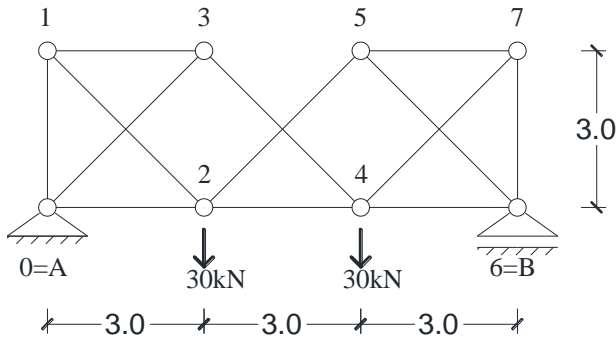
Z_n – uticaj Z na zamenjujućem nosaču usled reakcije $X_n = 1$

Uticaji uklonjenih elemenata zamenjuju se njihovim reakcijama:



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Zadatak: Za rešetku i opterećenje prema skici odrediti sile u štapovima.



2 uklonjena elementa: X_1, X_2
2 dodata elementa: Y_1, Y_2

I varijantno rešenje – Jednačine sistema

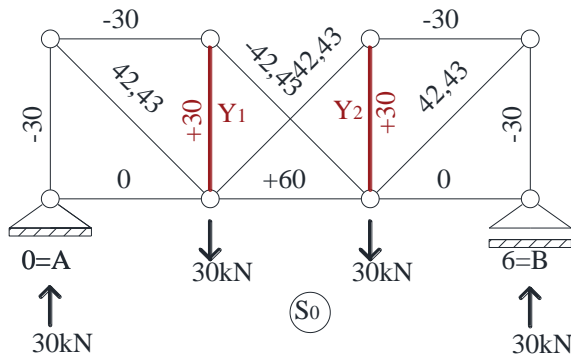
$$\begin{aligned} Y_{10} + Y_{11}X_1 + Y_{12}X_2 &= 0 \\ Y_{20} + Y_{21}X_1 + Y_{22}X_2 &= 0 \end{aligned}$$



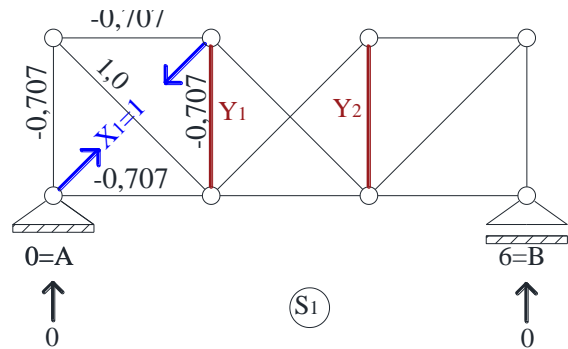
$$\begin{aligned} 30 - \sqrt{2}/2X_1 + 0X_2 &= 0 \rightarrow X_1 = 30\sqrt{2} \\ 30 + 0X_1 - \sqrt{2}/2X_2 &= 0 \rightarrow X_2 = 30\sqrt{2} \end{aligned}$$

-Sile u zamjenjujućem nosaču

*Zadato opterećenje

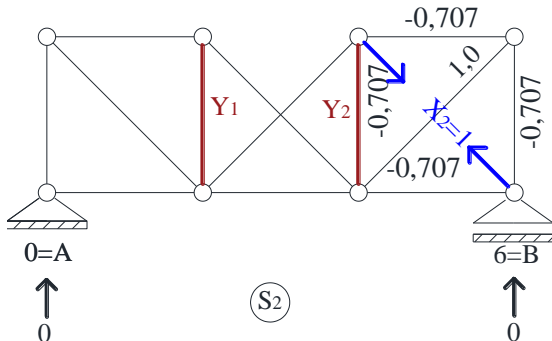


*Stanje $X_1=1,0$



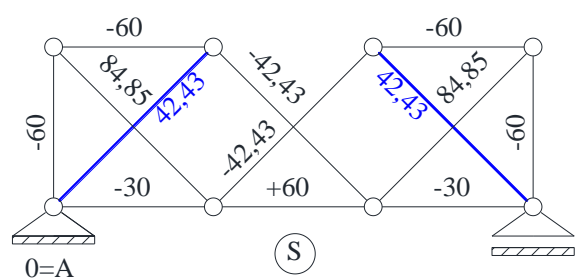
$$Y_{10} = 30 \text{ kN}; Y_{20} = 30 \text{ kN}; Y_{11} = -0,707; Y_{21} = 0$$

*Stanje $X_2=1,0$



$$Y_{12} = 0; Y_{22} = -0,707$$

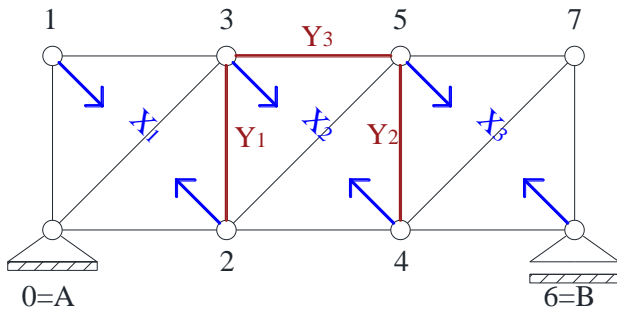
Uticaji na stvarnom nosaču



$$S = S_0 + S_1X_1 + S_2X_2$$

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II varijantno rešenje – Jednačine sistema



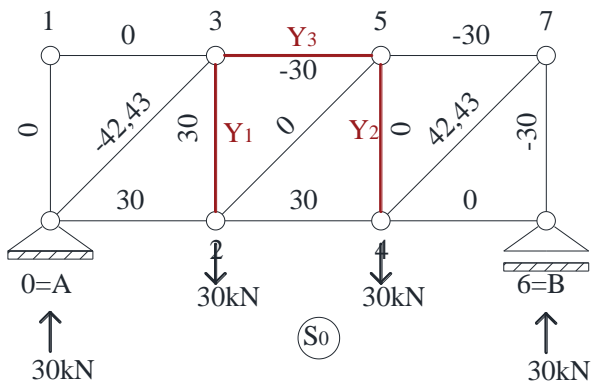
3 uklonjena elementa: X_1, X_2, X_3
 3 dodata elementa: Y_1, Y_2, Y_3

$$\begin{aligned} Y_{10} + Y_{11}X_1 + Y_{12}X_2 + Y_{13}X_3 &= 0 \\ Y_{20} + Y_{21}X_1 + Y_{22}X_2 + Y_{23}X_3 &= 0 \\ Y_{30} + Y_{31}X_1 + Y_{32}X_2 + Y_{33}X_3 &= 0 \end{aligned}$$

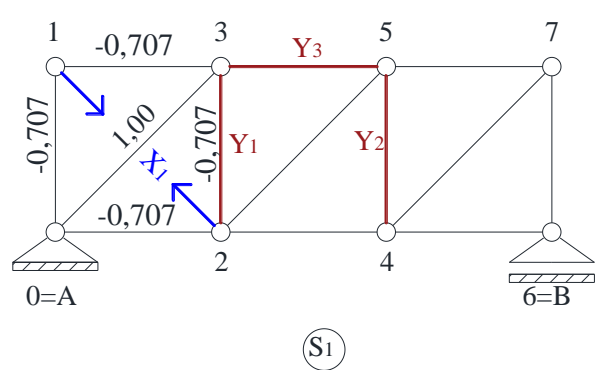
$$\begin{aligned} 30 - \sqrt{2}/2 X_1 - \sqrt{2}/2 X_2 + 0 X_3 &= 0 \rightarrow X_1 = 60\sqrt{2} \\ 0 + 0 X_1 - \sqrt{2}/2 X_2 - \sqrt{2}/2 X_3 &= 0 \rightarrow X_2 = -30\sqrt{2} \\ -30 + 0 X_1 - \sqrt{2}/2 X_2 + 0 X_3 &= 0 \rightarrow X_3 = 30\sqrt{2} \end{aligned}$$

-Sile u zamjenjujućem nosaču

*Zadato opterećenje

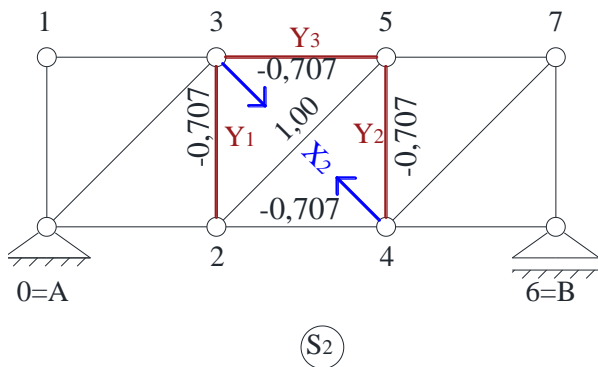


*Stanje $X_1=1,0$

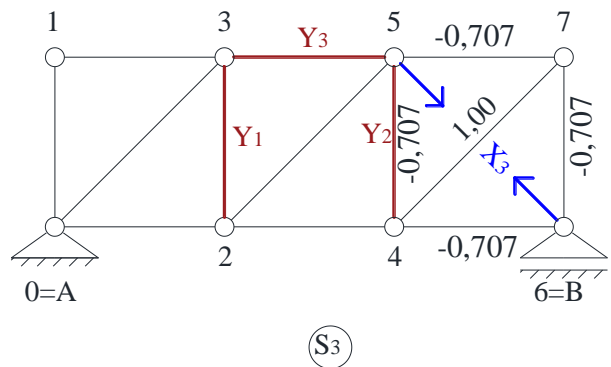


$$Y_{10} = 30\text{kN}; Y_{20} = 0; Y_{30} = -30\text{kN}; Y_{11} = -0,707; Y_{21} = 0; Y_{31} = 0$$

*Stanje $X_2=1,0$

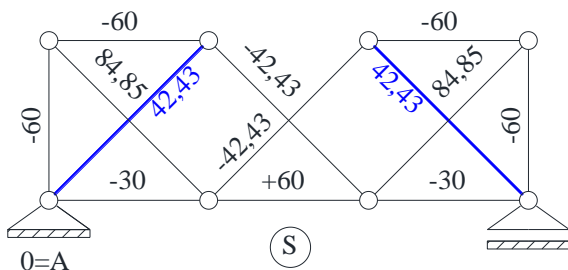


*Stanje $X_3=1,0$



$$Y_{12} = -0,707; Y_{22} = -0,707; Y_{32} = -0,707$$

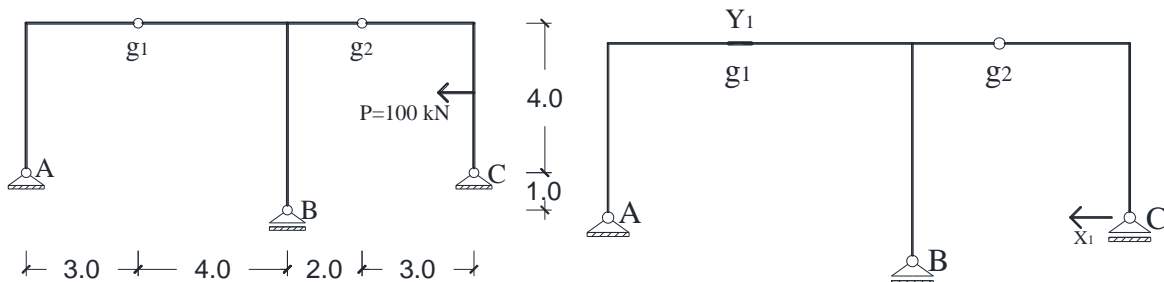
$$Y_{13} = 0; Y_{23} = -0,707; Y_{33} = 0$$



Uticaji na stvarnom nosaču
 $S = S_0 + S_1 X_1 + S_2 X_2 + S_3 X_3$

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Zadatak: Za nosač i opterećenje prema skici odrediti dijagrame presečnih sila.
-Zamjenjujući nosač

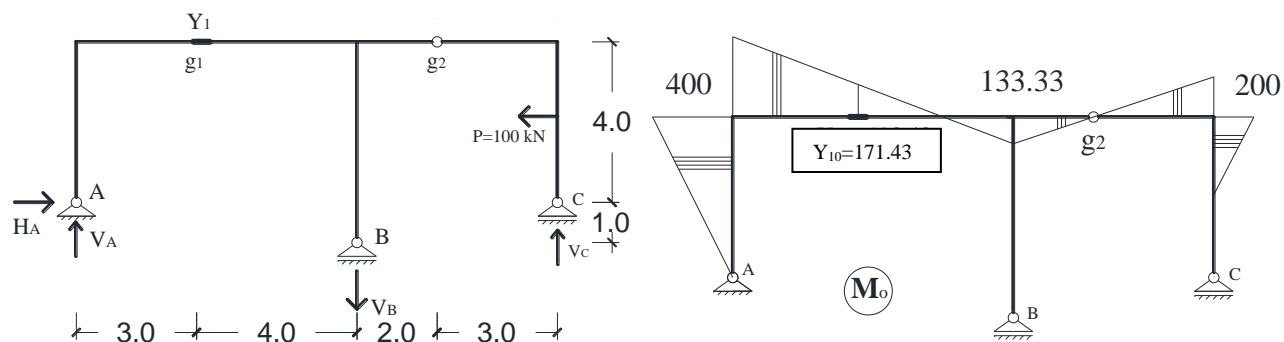


-Jednačina sistema

$$Y_{10} + Y_{11}X_1 = 0$$

-Reakcije oslonaca i presečne sile u zamjenjujućem nosaču:

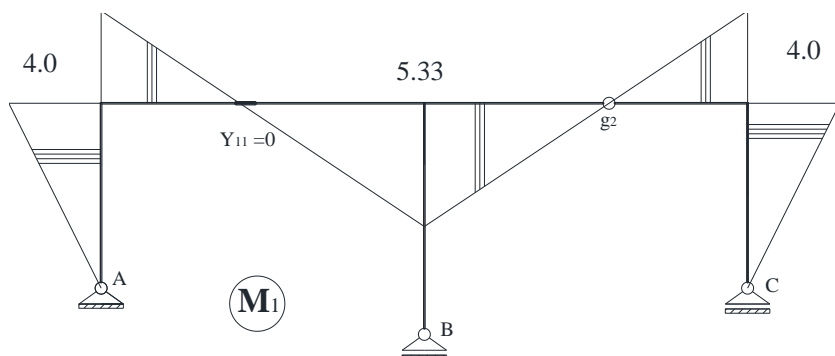
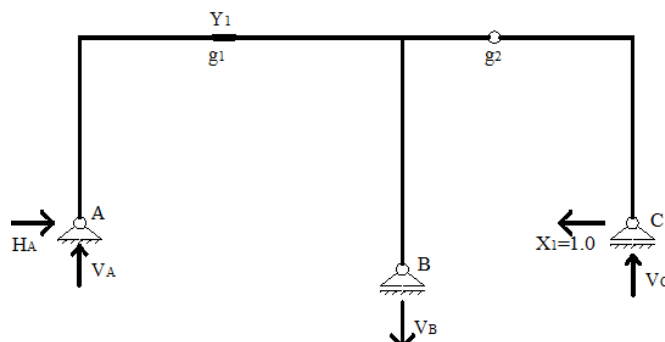
*Spoljašnje opterećenje



1. $\sum M_{g2}^d = 0 \rightarrow V_C = \frac{1}{3} 100 \cdot 2 = 66.67 \text{ kN}$
2. $\sum M_A = 0 \rightarrow V_B = \frac{1}{7} (66.67 \cdot 12 + 100 \cdot 2) = 142.86 \text{ kN}$
3. $\sum V_i = 0 \rightarrow V_A = 142.86 - 66.67 = 76.19 \text{ kN}$
4. $\sum H_i = 0 \rightarrow H_A = 100 \text{ kN}$

*Stanje $X_1=1.0$

1. $\sum M_{g2}^d = 0 \rightarrow V_C = \frac{1}{3} 1 \cdot 4 = 1.33$
2. $\sum M_A = 0 \rightarrow V_B = \frac{1}{7} (1.33 \cdot 12) = 2.285$
3. $\sum V_i = 0 \rightarrow V_A = 2.285 - 1.33 = 0.952$
4. $\sum H_i = 0 \rightarrow H_A = 1.0$



$$Y_{10} + Y_{11}X_1 = 0$$

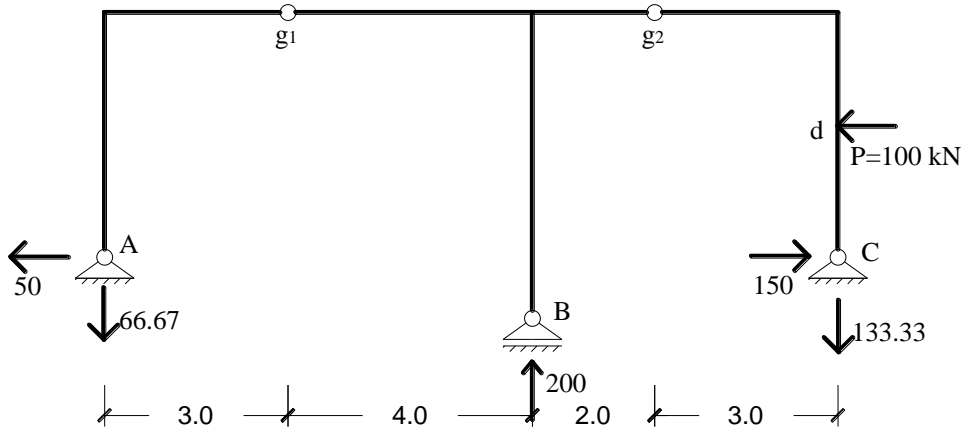
$$-171.43 - 1.143X_1 = 0$$

$$X_1 = -\frac{171.43}{1.143} = -150.0 \text{ kN}$$

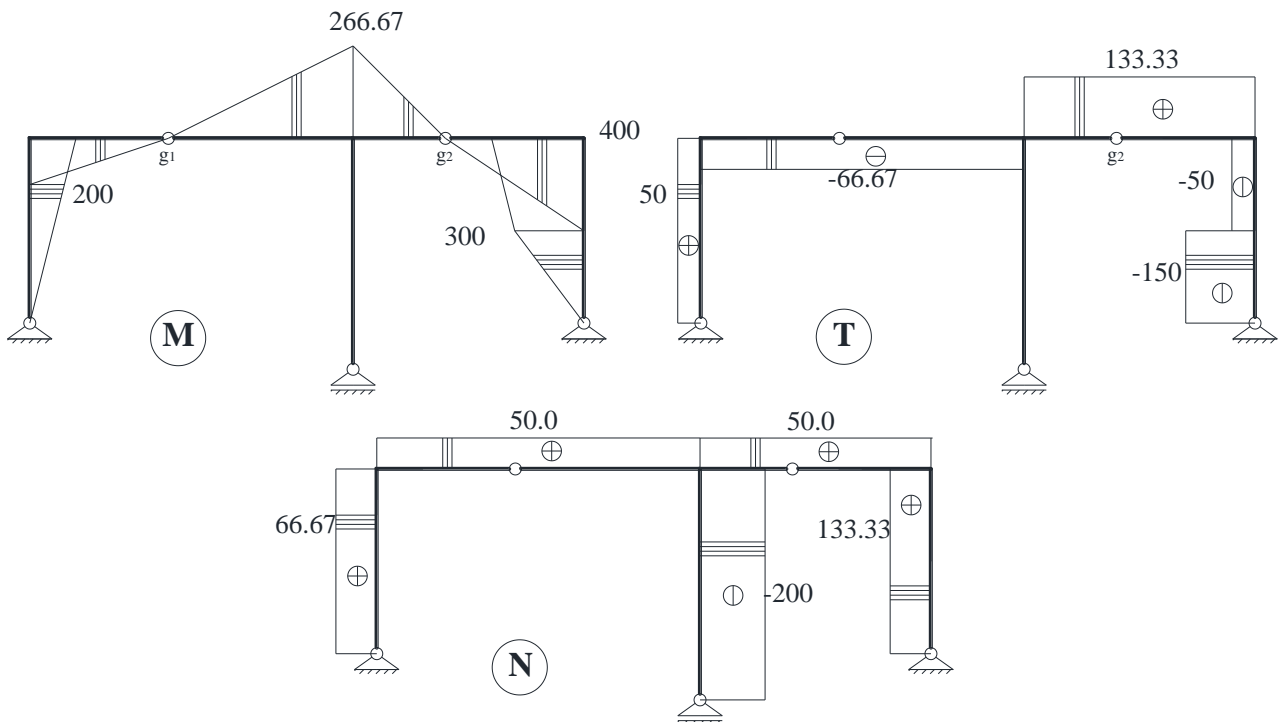
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-Reakcije oslonaca na stvarnom nosaču

$$R = R_{10} + R_{11}X_1 \rightarrow V_C = -133.33 \text{ kN}; V_B = -200 \text{ kN}; V_A = -66.67 \text{ kN}; H_A = -50.0 \text{ kN}$$



-Dijagram presječnih sila na stvarnom nosaču



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Metod zamene elemenata- uticajne linije

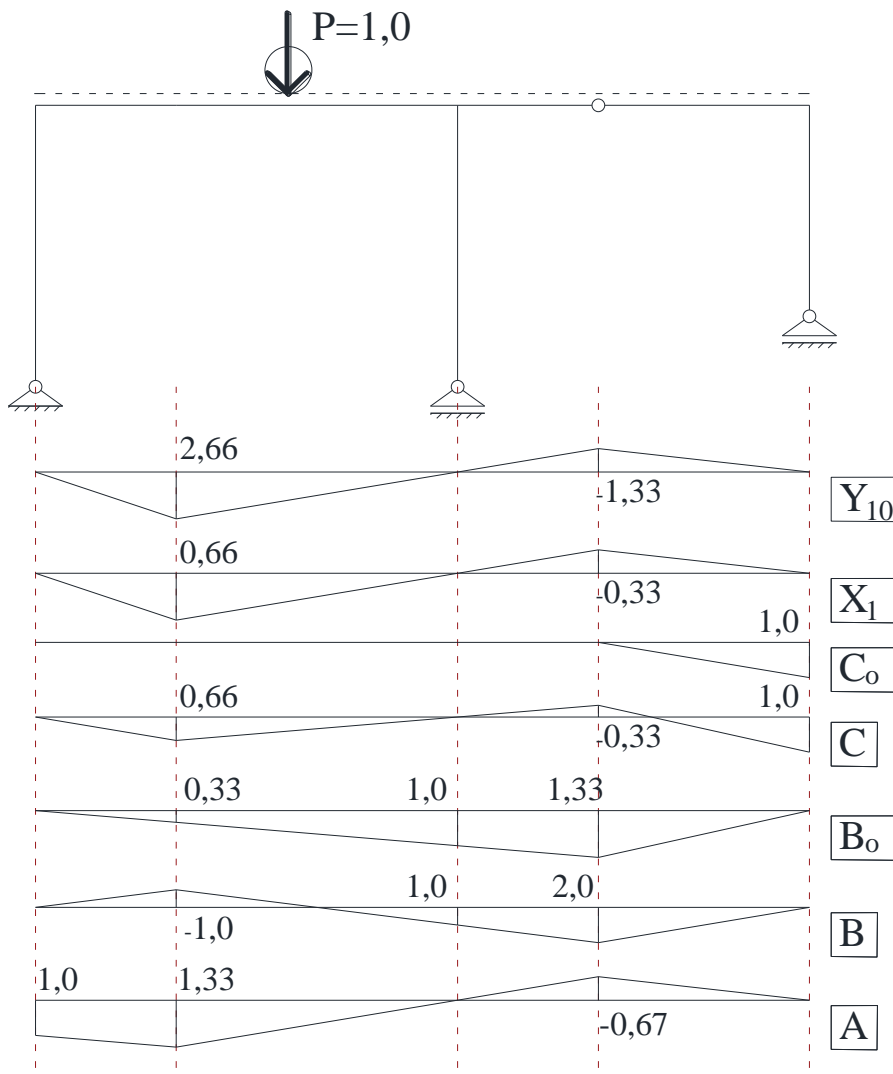
$$Z = Z_0 + Z_1X_1 + Z_2X_2 + \dots + Z_nX_n$$

Z – uticajna linija u zadatom nosaču (reakcije oslonaca, presečne sile)

Z_0 - uticajna linija na zamenjujućem nosaču usled pokretne jedinične koncentrisane sile

Z_n –vrijednost uticaja Z na zamenjujućem nosaču usled stanja $X_n = 1$

$$Y_{10} + Y_{11}X_1 = 0$$



$$X_1 = -\frac{Y_{10}}{Y_{11}} = 0,25Y_{10}, \quad Y_{11} = -4 \text{ (vrednost se očitava sa dijagrama } M_1)$$

$$C_0 = \frac{u_{g2}}{6}, \quad C = C_0 + X_1 \cdot C_1, \quad C_1 = 1,0$$

$$\sum M_A = 0 \rightarrow B_0 = \frac{u_A - 22C_0}{12}, \quad B = B_0 + X_1 \cdot B_1, \quad B_1 = -2,0$$

$$A = P - B - C$$