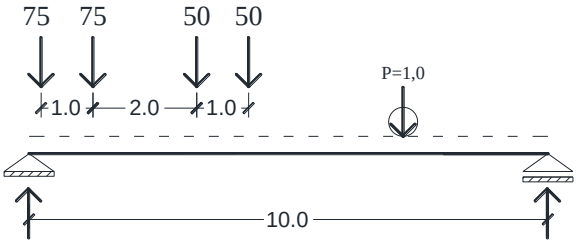
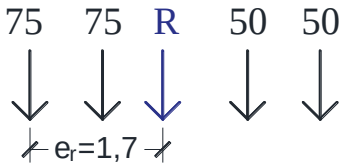
Zadatak: *Za prostu gredu prema skici odrediti dijagrame ekstremnih vrednosti momenata savijanja (anvelopa) usled datog pokretnog sistema vezanih koncentrisanih sila.*



Rezultanta:

$$R_m = \sum_{m=1}^{n} P_m = 75 + 75 + 50 + 50 = 250kN$$

-Lokalni položaj rezultante



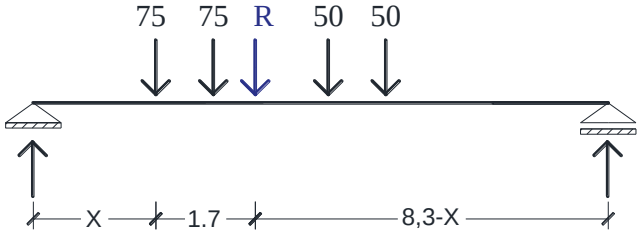
$$e_r = \frac{75 \cdot 1 + 50 \cdot 3 + 50 \cdot 4}{250} = 1{,}7m$$

-Dužine segmenata u kojima su određene sile mjerodavne

$$\lambda_m = \frac{P_m}{R} l, \qquad \lambda_1 = \lambda_2 = \frac{75}{250} 10 = 3m, \qquad \lambda_3 = \lambda_4 = \frac{50}{250} 10 = 2m.$$

*Segment I (Anvelopa od 0-3m – Segment u kome je mjerodavna P_1)

$$0 \le X_1 \le 3$$



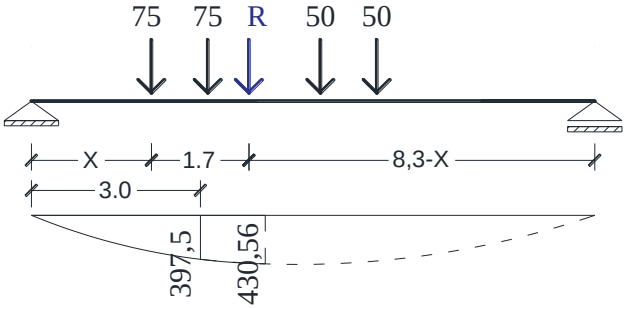
$$\sum M_B = 0 \rightarrow A \cdot l = R(8{,}3 - X)$$
$$A = \frac{250}{10}(8{,}3 - X) = 207{,}5 - 25X$$
$$M_x = A \cdot X = 207{,}5X - 25X^2$$

U ekstremnoj vrednosti momenta imamo horizontalnu tangentu čiji je izvod =0

Mjesto na kome imamo ekstremni moment:

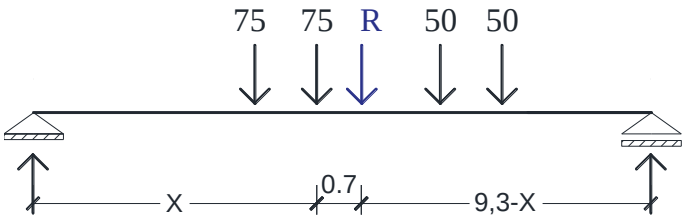
$$\frac{dM}{dx} = 0 \rightarrow 207{,}5 - 50X = 0 \rightarrow X = 4{,}15m$$

$$M_1 = \begin{cases} X = 0 \rightarrow M_1 = 0 \\ X = 3 \rightarrow M_1 = 397{,}5\ kNm \\ X = 4{,}15 \rightarrow M_1 = 430{,}56\ kNm \end{cases}$$



*Segment II (Anvelopa od 3-6m – Segment u kome je mjerodavna P_2)

$$3 \le X_2 \le 6$$

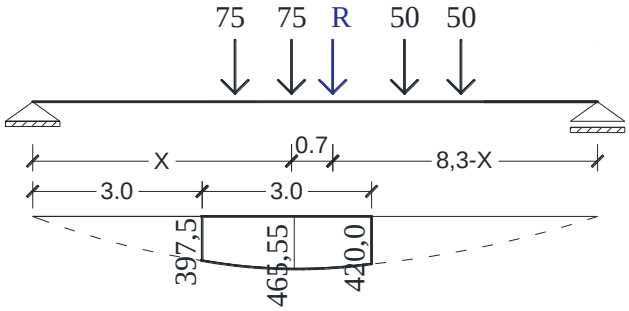


$$\sum M_B = 0 \rightarrow A \cdot l = R(9{,}3 - X)$$
$$A = \frac{250}{10}(9{,}3 - X) = 232{,}5 - 25X$$
$$M_x = A \cdot X - P \cdot 1$$
$$= 232{,}5X - 25X^2 - 75$$

U ekstremnoj vrednosti momenta imamo horizontalnu tangentu čiji je izvod =0
Mjesto na kome imamo ekstremni moment:

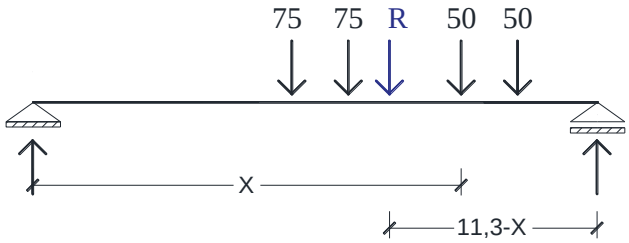
$$\frac{dM}{dx} = 0 \rightarrow 232{,}5 - 50X = 0 \rightarrow X = 4{,}65m$$

$$M_2 = \begin{cases} X = 3 \rightarrow M_2 = 397{,}5\ kNm \\ X = 4{,}65 \rightarrow M_2 = 465{,}55\ kNm \\ X = 6 \rightarrow M_2 = 420{,}0\ kNm \end{cases}$$



*Segment III (Anvelopa od 6-8m – Segment u kome je mjerodavna P_3)

$$6 \le X_3 \le 8$$

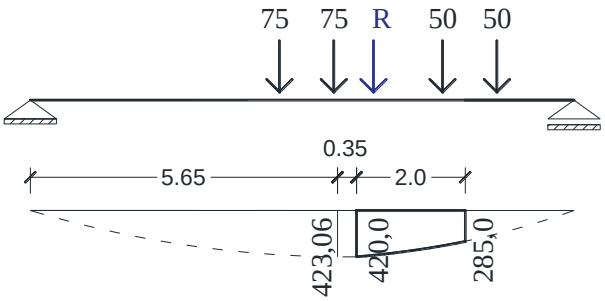


$$\sum M_B = 0 \rightarrow A \cdot l = R(11{,}3 - X)$$
$$A = \frac{250}{10}(11{,}3 - X) = 282{,}5 - 25X$$
$$M_x = A \cdot X - P \cdot 3 - P \cdot 2 =$$
$$= 282{,}5X - 25X^2 - 375$$

U ekstremnoj vrednosti momenta imamo horizontalnu tangentu čiji je izvod =0
Mjesto na kome imamo ekstremni moment:

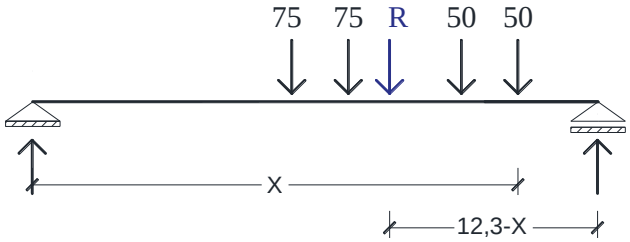
$$\frac{dM}{dx} = 0 \rightarrow 282{,}5 - 50X = 0 \rightarrow X = 5{,}65m$$

$$M_3 = \begin{cases} X = 5{,}65 \rightarrow M_3 = 423{,}06\ kNm \\ X = 6 \rightarrow M_3 = 420{,}0\ kNm \\ X = 8 \rightarrow M_3 = 285{,}0\ kNm \end{cases}$$



*Segment IV (Anvelopa od 8-10m – Segment u kome je mjerodavna P_4)

$$8 \leq X_4 \leq 10$$



$$\sum M_B = 0 \rightarrow A \cdot l = R(12{,}3 - X)$$

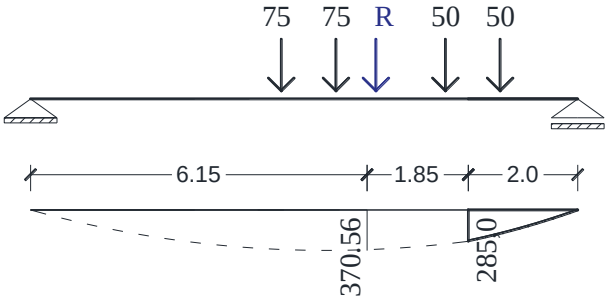
$$A = \frac{250}{10}(12{,}3 - X) = 307{,}5 - 25X$$

$$M_x = A \cdot X - P \cdot 4 - P \cdot 3 - P \cdot 1 =$$
$$= 307{,}5X - 25X^2 - 575$$

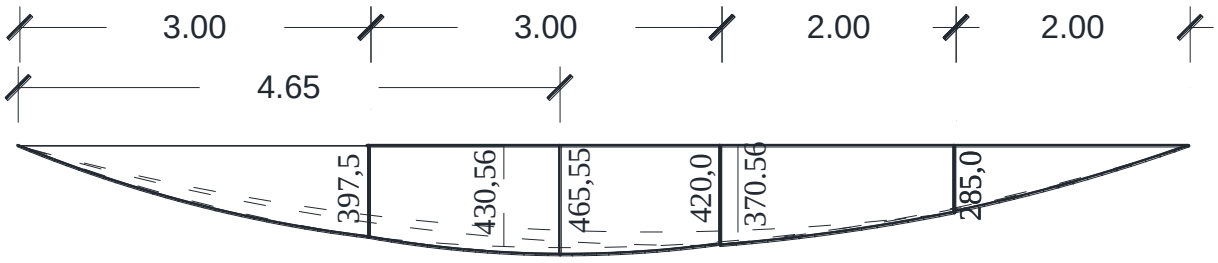
U ekstremnoj vrednosti momenta imamo horizontalnu tangentu čiji je izvod =0
Mjesto na kome imamo ekstremni moment:

$$\frac{dM}{dx} = 0 \rightarrow 307{,}5 - 50X = 0 \rightarrow X = 6{,}15m$$

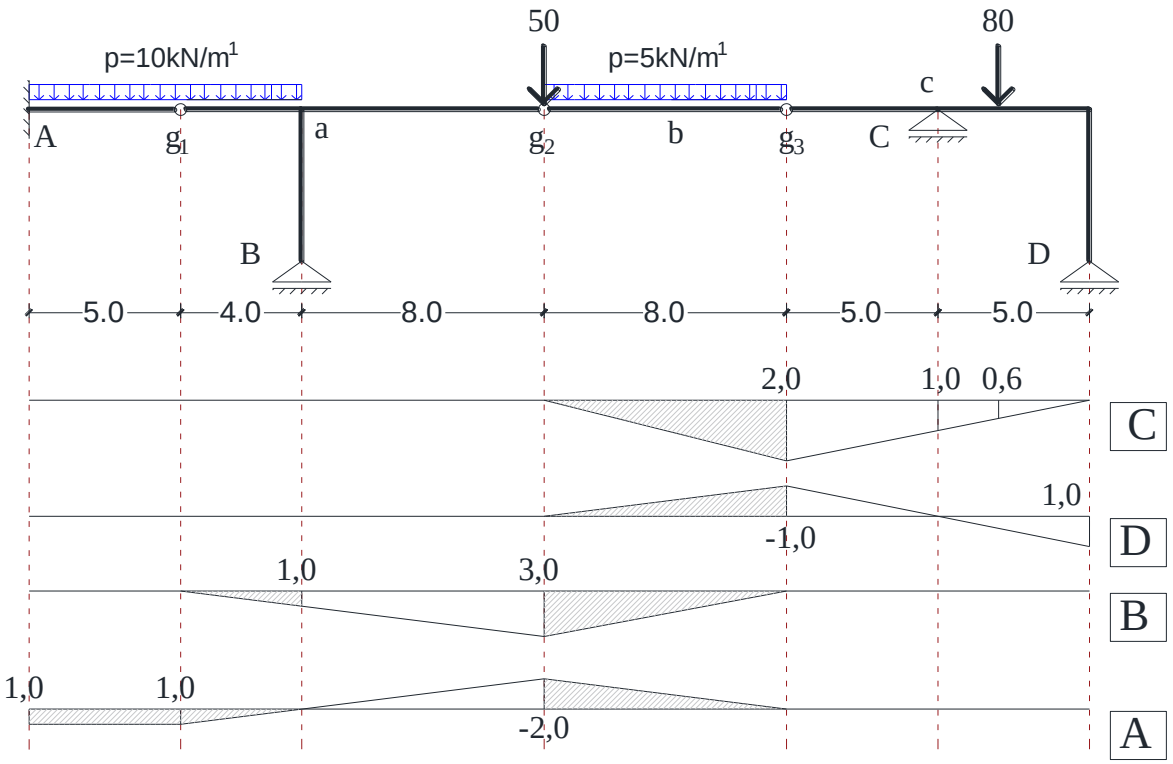
$$M_4 = \begin{cases} X = 6{,}15 \rightarrow M_4 = 370{,}56\ kNm \\ X = 8 \rightarrow M_4 = 285{,}0\ kNm \\ X = 10 \rightarrow M_4 = 0 \end{cases}$$



Anvelopa na cijelom nosaču



Zadatak: *Za nosač sa slike izračunati reakcije oslonaca i presečne sile u presečima „a“, „b“ i „c“ pomoću uticajnih linija.*



$$\sum {M_{g_3}}^{d} = 0 \rightarrow D \cdot 10 + C \cdot 5 - P \cdot u_{g3} = 0 \rightarrow \boldsymbol{D = \frac{u_{g3}}{10} - 0,5C}$$

$$\sum {M_{g_2}}^{d} = 0 \rightarrow D \cdot 18 + C \cdot 13 - P \cdot u_{g2} = 0 \rightarrow \boldsymbol{C = 0,25 \cdot u_{g2} - 0,45 u_{g3}}$$

$$\sum {M_{g_1}}^{d} = 0 \rightarrow D \cdot 30 + C \cdot 25 + B \cdot 4 - P \cdot u_{g1} = 0 \rightarrow \boldsymbol{B = \frac{u_{g1}}{4} - 7,5D - 6,25C}$$

$$\sum V_i = 0 \rightarrow A + B + C + D - P = 0 \rightarrow \boldsymbol{A = 1 - B - C - D}$$

$$A = (1 \cdot 5 + 1 \cdot 4/2)10 - \frac{2}{2}8 \cdot 5 - 50 \cdot 2 = -70\ kN$$

$$B = 1 \cdot \frac{4}{2} \cdot 10 + \frac{3}{2}8 \cdot 5 + 50 \cdot 3 = 230\ kN$$

$$C = \frac{2}{2}8 \cdot 5 + 80 \cdot 0,6 = 88\ kN$$

$$D = -\frac{1}{2}8 \cdot 5 + 80 \cdot 0,4 = 12\ kN$$

$$\sum {M_{g_1}}^{l} = 0 \rightarrow A \cdot 5 - P \cdot u_{g1} + M_A = 0 \rightarrow$$

$$\boldsymbol{M_A = u_{g1} - A \cdot 5} \qquad \boldsymbol{M_A} = -\frac{5}{2}9 \cdot 10 + \frac{10}{2}8 \cdot 5 + 10 \cdot 50 = 475 kNm$$

$$\boldsymbol{M_a = M_A - u_a + A \cdot 9} \qquad \boldsymbol{M_a} = -\frac{8}{2}8 \cdot 5 - 8 \cdot 50 = -560 kNm$$

$$\boldsymbol{M_b = 14 \cdot D + 9 \cdot C - u_b} \qquad \boldsymbol{M_b} = \frac{2}{2}8 \cdot 5 = 40 kNm$$

$\boldsymbol{M_c = 5 \cdot D - u_c}$

$$M_c = -\frac{5}{2} 8 \cdot 5 = -100\ kNm$$

$\boldsymbol{T_a{}^l = A - P}$

$\boldsymbol{T_a{}^D = A - P + B}$

$\boldsymbol{T_b = A - P + B}$

$\boldsymbol{T_c{}^l = A - P + B}$

$\boldsymbol{T_c{}^d = P - D}$

$$T_a{}^l = -\frac{1}{2} 4 \cdot 10 - \frac{2}{2} 8 \cdot 5 - 2 \cdot 50 = -160kN$$

$$T_a{}^d = \frac{1}{2} 8 \cdot 5 + 1 \cdot 50 = 70kN$$

$$T_b = 0$$

$$T_c{}^l = -\frac{1}{2} 8 \cdot 5 = -20kN$$

$$T_c{}^d = \frac{1}{2} 8 \cdot 5 + 0{,}6 \cdot 80 = 68\ kN$$