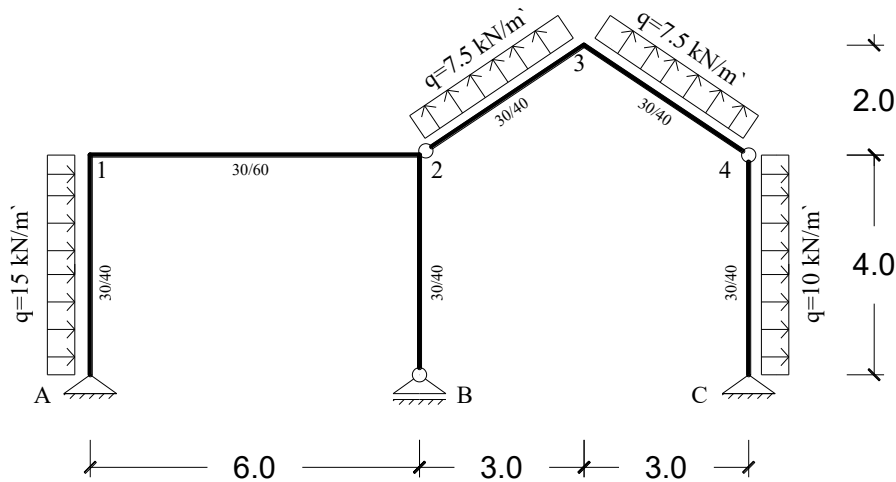
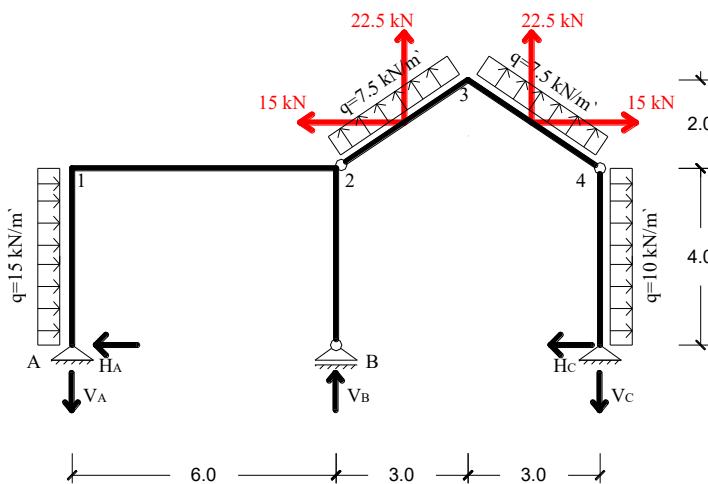


1. Za nosač na slici odrediti dijagram vertikalnog pomeranja poteza 1-2-3-4, sa ordinatama na svakih 1.5m, usled jednovremenog delovanja datog opterećenja i temperature u osama štapova A-1, B-2 ($t^0 = +25^{\circ}\text{C}$). Geometrijske karakteristike poprečnih presjeka date su na slici. Uticaj normalnih sila na deformaciju zanemariti.

$$E = 3 \cdot 10^7 \text{ kN/m}^2, \quad \alpha_t = 10^{-5} \frac{1}{^{\circ}\text{C}}$$



a) Reakcije oslonaca i dijagrami presečnih sila



$$1. \sum M_4^{dole} = 0 \rightarrow -H_c \cdot 4 + 10 \cdot 4 \cdot 2 = 0 \rightarrow H_c = 20 \text{ kN}$$

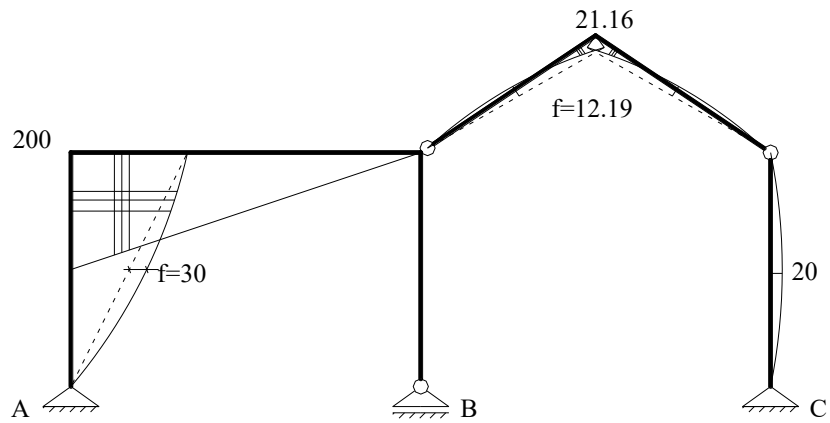
$$2. \sum M_2^{desno} = 0 \rightarrow -V_c \cdot 6 - 20 \cdot 4 + 10 \cdot 4 \cdot 2 + 22.5 \cdot (4.5 + 1.5) = 0 \rightarrow V_c = 22.5 \text{ kN}$$

$$3. \sum H_i = 0 \rightarrow H_A = 80 \text{ kN}$$

$$4. \sum M_2^{levo} = 0 \rightarrow -V_A \cdot 6 + 80 \cdot 4 - 15 \cdot 4 \cdot 2 \rightarrow V_A = 33.33 \text{ kN}$$

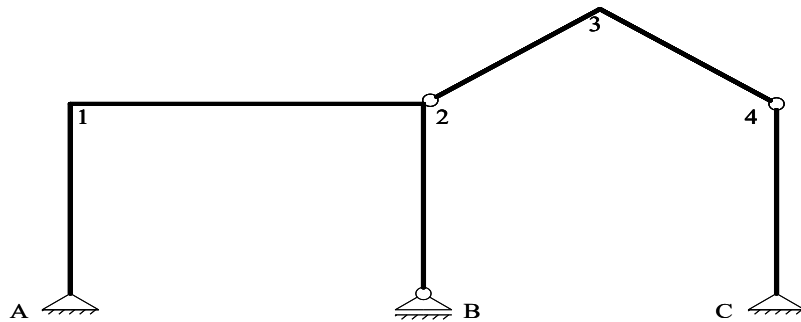
$$5. \sum V_i = 0 \rightarrow V_B = 10.83 \text{ kN}$$

- dijagram momenta savijanja (M [kNm])



- Fiktivni nosač

$v \neq 0$	$v \neq 0$	$v \neq 0$	$v = 0$
$\varphi_t \neq 0$	$\varphi_t^l \neq \varphi_t^d \neq 0$	$\varphi_t^l = \varphi_t^d \neq 0$	$\varphi_t \neq 0$

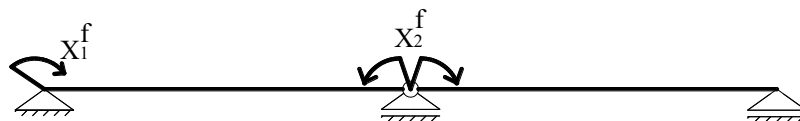


$M^f \neq 0$	$M^f \neq 0$	$M^f \neq 0$	$M^f = 0$
$T^f \neq 0$	$T^{f^l} \neq T^{f^d} \neq 0$	$T^{f^l} = T^{f^d} \neq 0$	$T^f \neq 0$

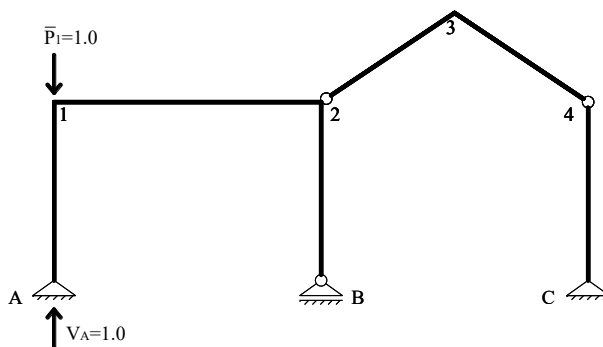


$$n = Z_s + Z_k + Z_o + Z_u - 2K = 2 + 1 + 4 + 1 - 2 \cdot 3 = 2 \text{ x stat. neodređen fiktivni nosač}$$

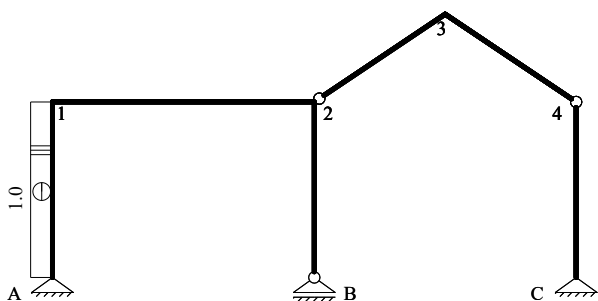
-Osnovni sistem datog statički neodređenog fiktivnog nosača



- Statički neodređena fiktivna veličina X_1^f

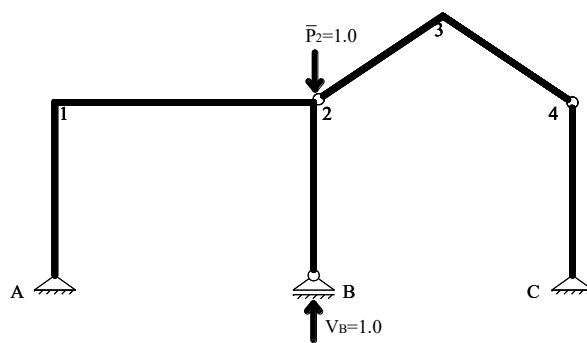


- Dijagram normalnih sila usled generalisane sile (\bar{N})

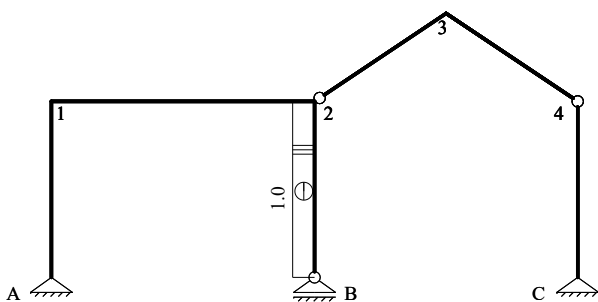


$$X_1^f = \int \bar{N} \alpha_t t^0 ds = -1 \cdot 25 \cdot 10^{-5} \cdot 4 = -1 \cdot 10^{-3}$$

- Statički neodređena fiktivna veličina X_2^f



- Dijagram normalnih sila usled generalisane sile (\bar{N})

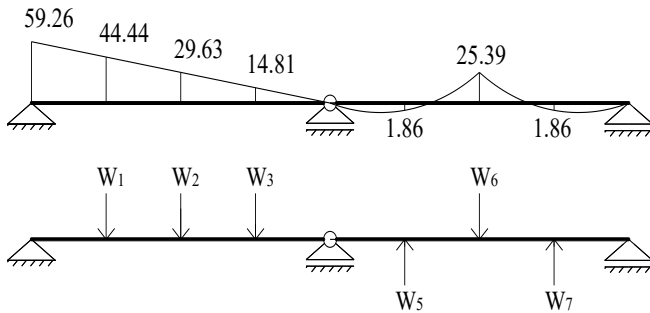


$$X_{12}^f = \int \bar{N} \alpha_t t^0 ds = -1 \cdot 25 \cdot 10^{-5} \cdot 4 = -1 \cdot 10^{-3}$$

- Fiktivno opterećenje raspodeljenim silama

$$I_c = \frac{0.3 \cdot 0.4^3}{12} = 1.6 \cdot 10^{-3} m^4, \quad EI_c = 48000 kNm^2$$

$$p^f = \left(\frac{M}{EI} + \alpha_t \frac{\Delta t^0}{h} \right) \frac{1}{\cos \alpha} \rightarrow EI_c p^f = \frac{I_c}{I} M \frac{1}{\cos \alpha}$$



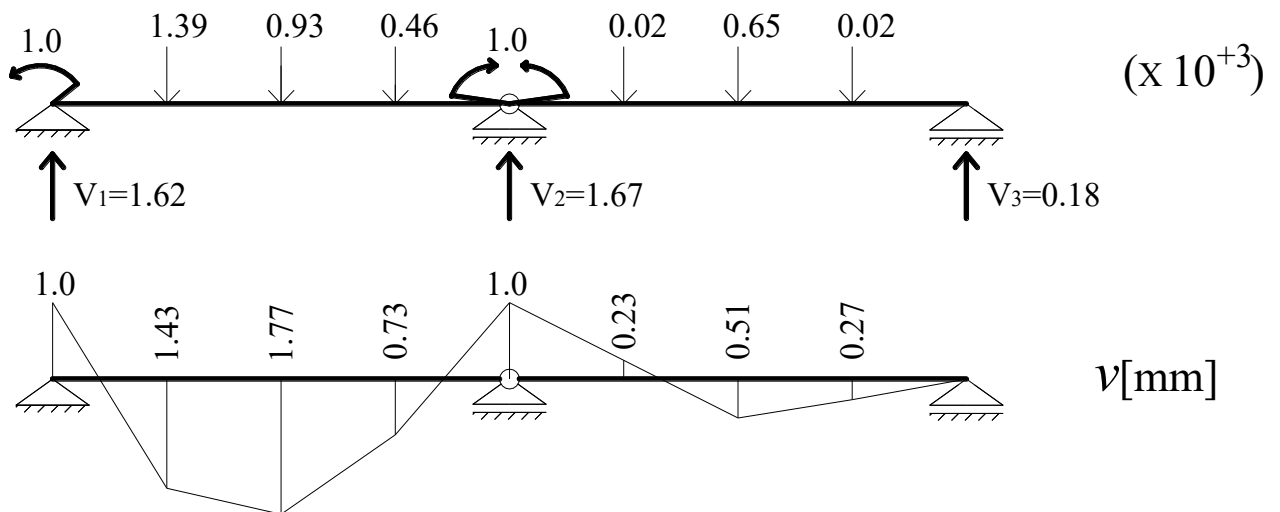
$$W_1 = \frac{1.5}{6} (59.26 + 4 \cdot 44.44 + 29.63) = 66.66$$

$$W_2 = \frac{1.5}{6} (44.44 + 4 \cdot 29.63 + 14.81) = 44.44$$

$$W_3 = \frac{1.5}{6} (29.63 + 4 \cdot 14.81 + 0) = 22.22$$

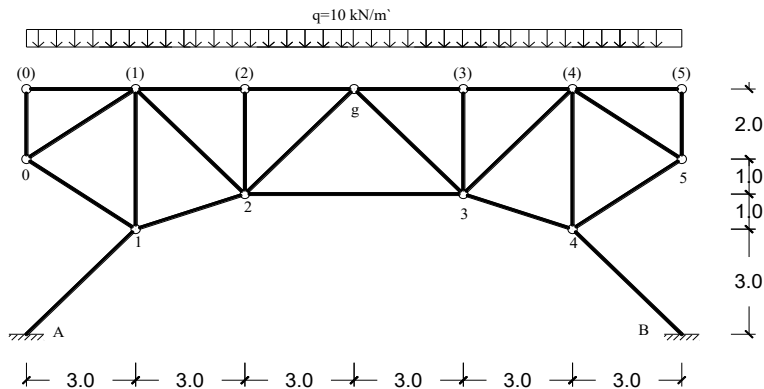
$$W_5 = W_7 = \frac{1.5}{12} (0 + 10 \cdot 1.86 - 25.39) = -0.85$$

$$W_6 = \frac{1.5}{12} (-1.86 + 10 \cdot 25.39 - 1.86) = 31.27$$



2. Za nosač i opterećenje na slici odrediti reakcije oslonaca i dijagrame presečnih sila.

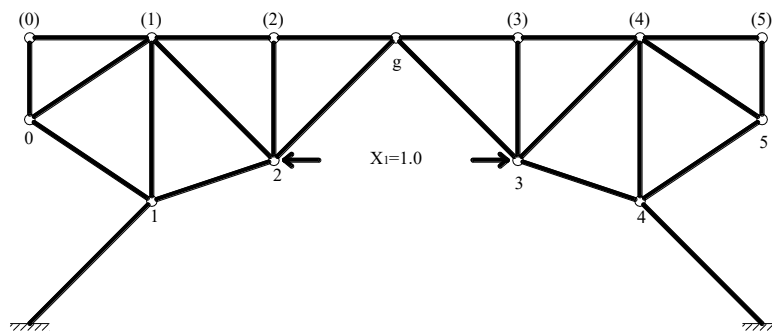
$$\frac{I}{F} = 0.03$$



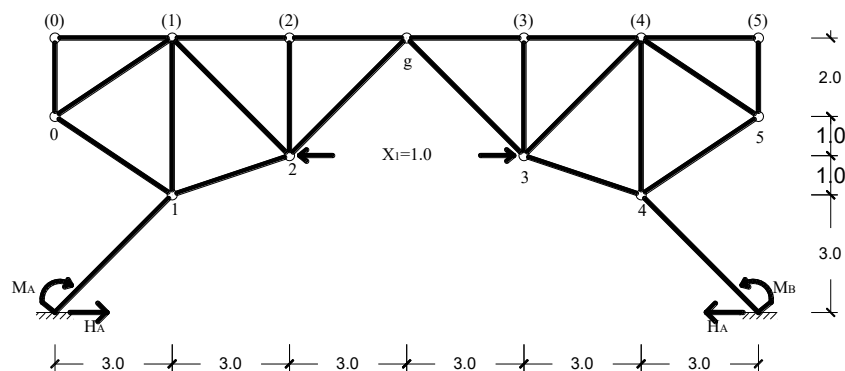
- Statička neodređenost nosača

$$n = Z_s + Z_k + Z_o + Z_u - 2K = 25 + 0 + 4 + 2 - 2 \cdot 15 = 1 \text{ x stat. neodređen nosač}$$

-Izbor osnovnog sistema



- Reakcije oslonaca usled statički nepoznate $X_1=1.0$

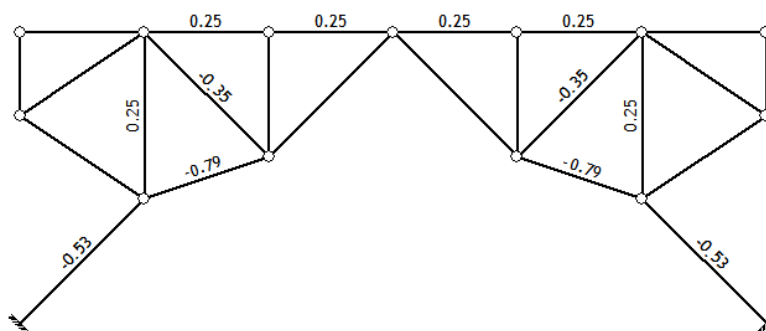


$$1. \sum M_1^{dole} = 0 \rightarrow M_A = 3H_A - 3V_A \rightarrow M_A = 2.25 = M_B$$

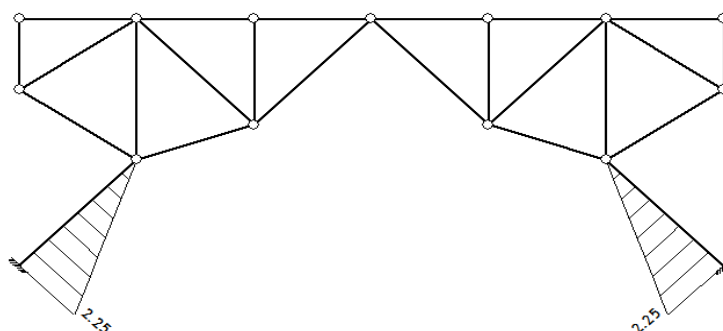
$$2. \sum M_4^{gore} = 0 \rightarrow V_A = 0 = V_B$$

$$3. \sum M_g^l = 0 \rightarrow H_A = 0.75 = H_B$$

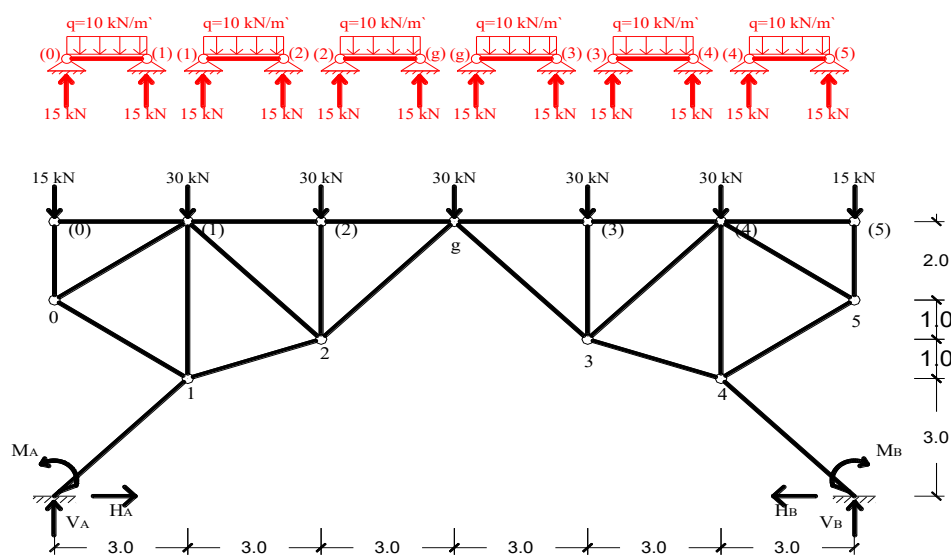
-Dijagram normalnih sila (N_1)



-Dijagram momenata savijanja (M_1)



- Reakcije oslonaca na osnovnom sistemu usled zadatog opterećenja

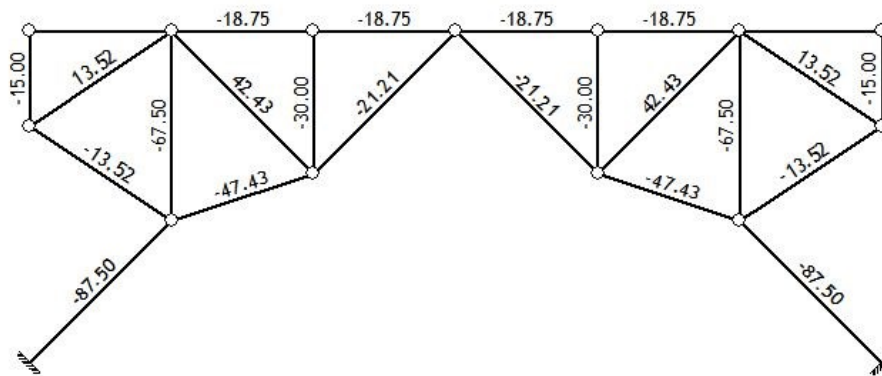


$$1. \sum M_1^{dole} = 0 \rightarrow M_A = -3H_A + 3V_A \rightarrow M_A = 168.75 \text{ kNm} = M_B$$

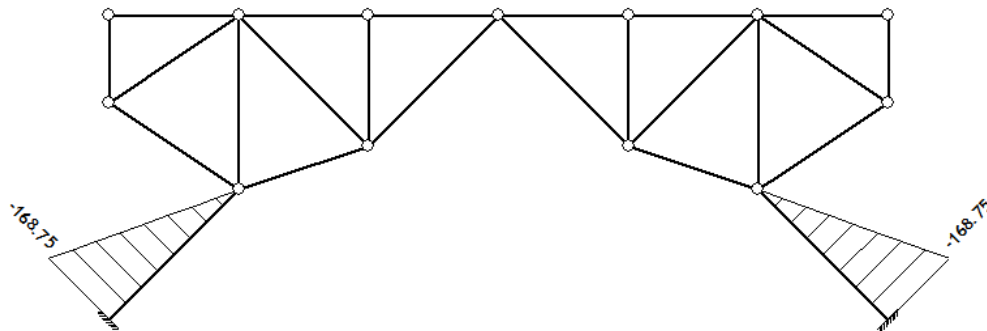
$$2. \sum M_4^{gore} = 0 \rightarrow V_A = 90 \text{ kN} = V_B$$

$$3. \sum M_g^l = 0 \rightarrow H_A = 33.75 \text{ kN} = H_B$$

-Dijagram normalnih sila usled zadatog opterećenja ($N_o[kN]$)



-Dijagram momenata savijanja usled zadatog opterećenja ($M_o[kN]$)



-Uslovna jednačina metode sila

$$\delta_{10} + \delta_{11}X_1 = 0$$

$$\begin{aligned} EI\delta_{10} &= \int M_1 M_0 ds + \frac{I}{F} \int N_1 N_0 \frac{F_c}{F} ds \\ &= 2 \left[-\frac{4,24}{3} 168,75 \cdot 2,25 + 0,03(3 \cdot (-0,25 \cdot 18,75 \cdot 2) - 4 \cdot 67,5 \cdot 0,25 \right. \\ &\quad \left. - 4,24(0,35 \cdot 42,43 - 0,53 \cdot 87,5) + 3,16 \cdot 0,79 \cdot 47,43) \right] = -1063,86 \end{aligned}$$

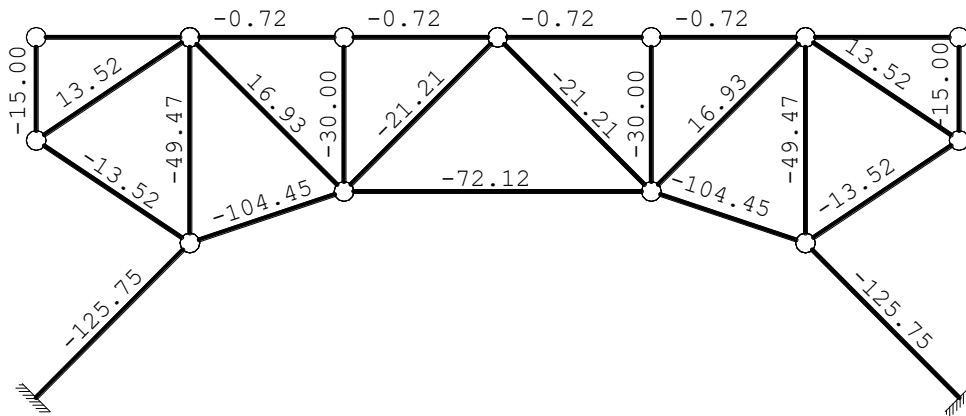
$$\begin{aligned} EI\delta_{11} &= \int M_1 M_1 ds + \frac{I}{F} \int N_1 N_1 ds \\ &= 2 \left[\frac{4,24}{3} 2,25 \cdot 2,25 \right. \\ &\quad \left. + 0,03(3 \cdot (0,25 \cdot 0,25 \cdot 2) + 4 \cdot 0,25 \cdot 0,25 + 4,24(0,35 \cdot 0,35 + 0,53 \cdot 53) + 3,16 \cdot 0,79 \cdot 0,79) \right] + 0,03 \cdot 1 \cdot 1 \cdot 6 = 14,75 \end{aligned}$$

-Uslovna jednačina metode sila

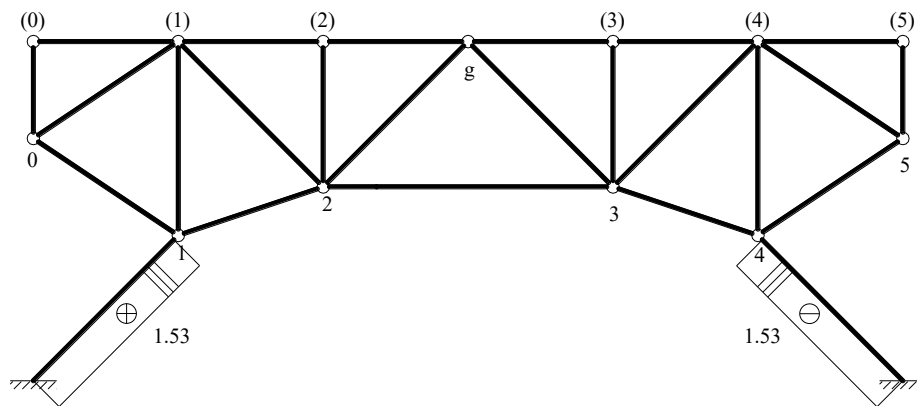
$$\delta_{10} + \delta_{11}X_1 = 0 \rightarrow X_1 = \frac{1063,86}{14,75} = 72,12 \text{ kN}$$

$$Z = Z_0 + Z_1X_1 \rightarrow H_A = H_B = 87,84 \text{ kN}, V_B = V_A = 90 \text{ kN}, M_B = M_A = 6,48 \text{ kNm}$$

- Dijagram normalnih sila na zadanom statički neodređenom nosaču (N[kN])



- Dijagram transverzalnih sila na zadanom statički neodređenom nosaču (T[kN])



- Dijagram momenata savijanja na zadanom statički neodređenom nosaču (M[kNm])

