Statički neodređeni rešetkasti nosač

Koeficijenti uz nepoznate:

$$\delta_{ij} = \sum \frac{S_i S_j}{EF} l \rightarrow EFc\delta_{ij} = \sum S_i S_j l'' , \qquad \delta_{io} = \sum \frac{S_i S_o}{EF} l \rightarrow EFc\delta_{ij} = \sum S_i S_0 l''$$

$$l'' = \frac{Fc}{F} l \qquad \delta_{it} = \sum S_i \alpha_t t° l \rightarrow EFc\delta_{ij} = EFc \sum S_i \alpha_t t° l$$

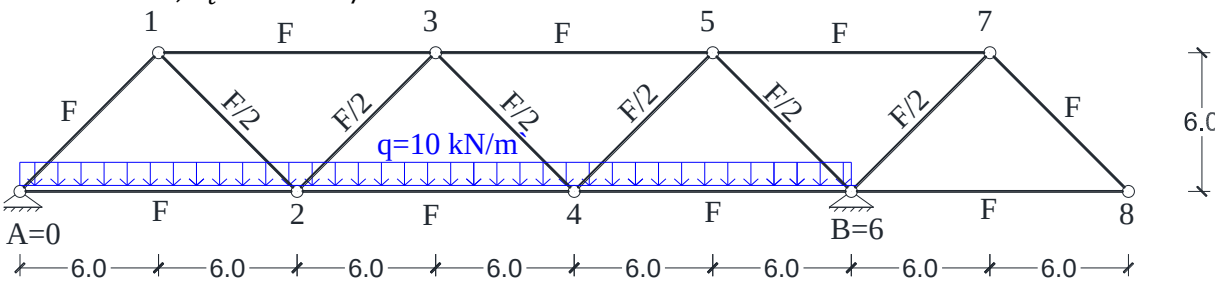
$$\delta_{ik} = \delta_{i0} + \delta_{it} + \delta_{ic} \qquad \delta_{ic} = -\sum C_i c_i \rightarrow EFc\delta_{ij} = -EFc \sum C_i c_i$$

Zadatak: *Za dati rešetkasti nosač prema skici odrediti vertikalno pomeranje čvora "8" usled:*

a) Datog opterećenja

b) Zagrevanja štapova gornjeg pojasa za 20K

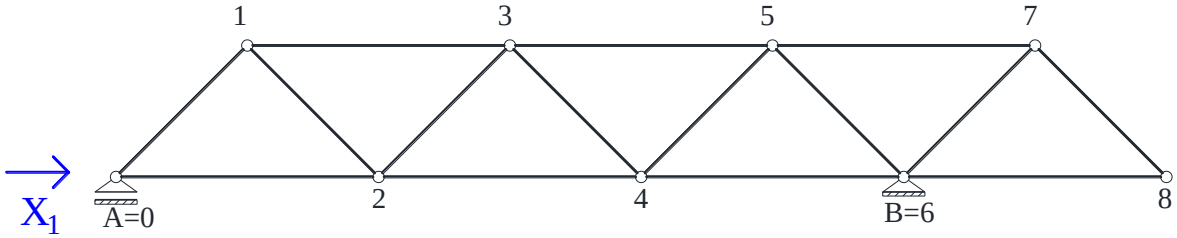
$EF = const., \alpha_t = 10^{-5}\, 1/K$



1. Statička neodređenpst

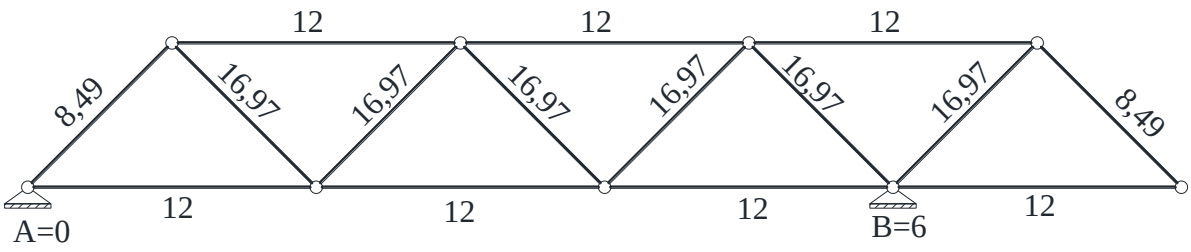
Zo=4, Zs=15, K=9, $n = Z_s + Z_o - 2K = 1 x stat. neod. nosač$

2. Osnovni sistem

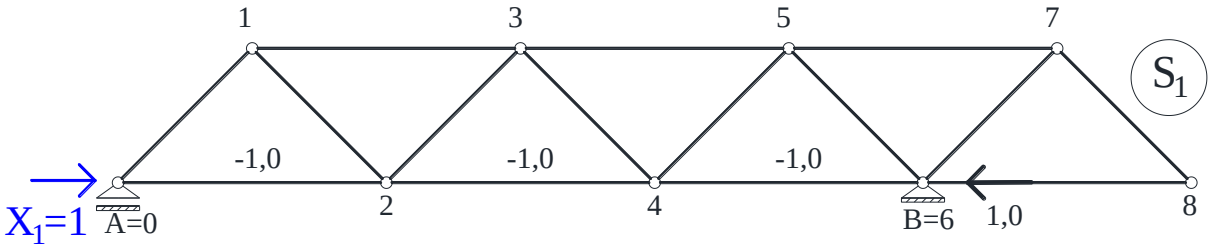


3. Redukovane dužine

$F_c = F, l'' = \frac{F_c}{F} l$



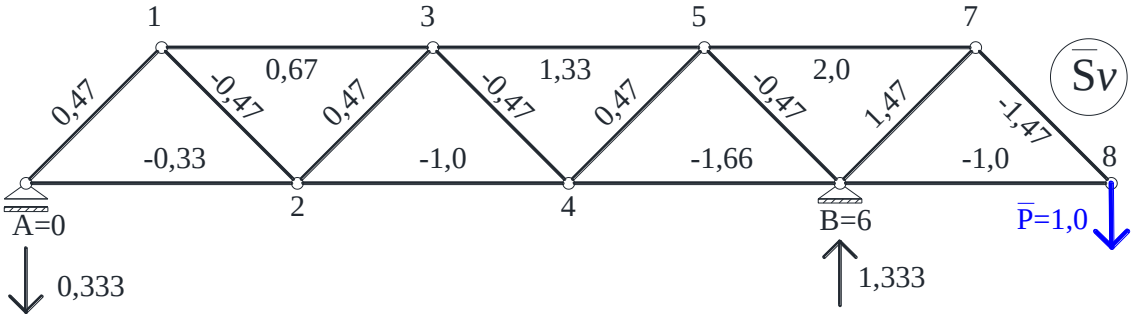
4. Sile u štapovima za stanje $X_1=1,0$



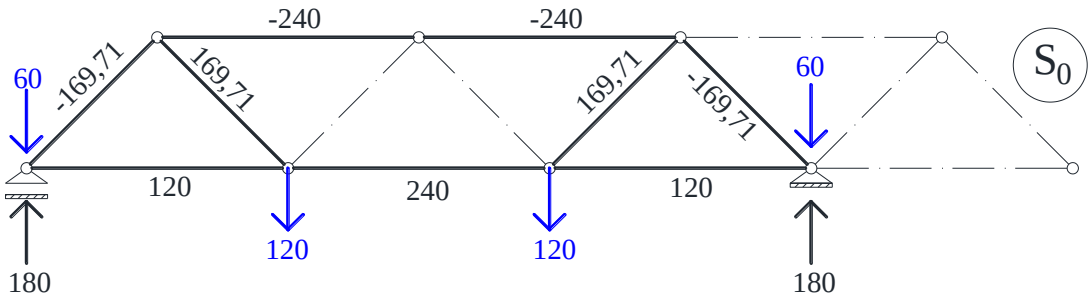
5. Koeficijenti uz nepoznate

$$EFc\delta_{11} = \sum S_1 S_1 l'' = 3 \cdot (-1) \cdot (-1) \cdot 12 = 36{,}0$$

6. Generalisana sila za vertikalno pomeranje



a) Pomeranje čvora "8" usled zadatog opterećenja



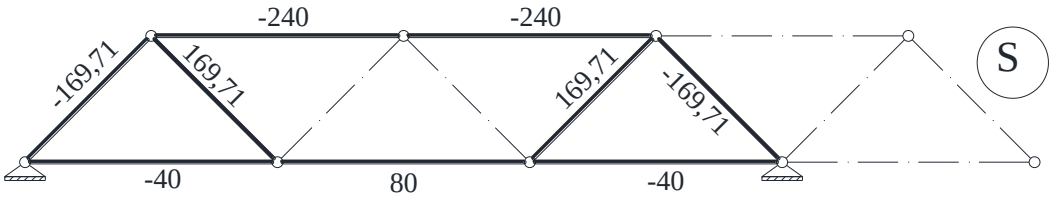
-Slobodni članovi

$$EFc\delta_{10} = \sum S_1 S_0 l'' = 2 \cdot (-1) \cdot 120 \cdot 12 + (-1) \cdot 240 \cdot 12 = -5760$$

-Statički neodređena veličina

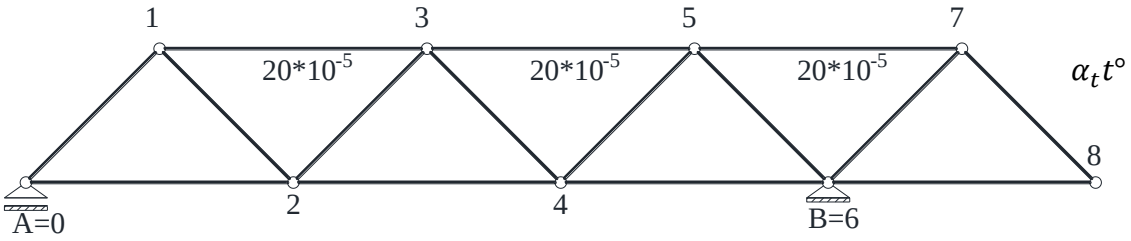
$$X_1 = -\frac{\delta_{10}}{\delta_{11}} = \frac{5760}{36} = 160\ kN$$

-Sile u štapovima $S = S_0 + S_1 X_1$



$$EFcv_8 = \sum S\bar{S}_v l''$$
$$= 12 \cdot [-240(0{,}67 + 1{,}33) - 40(-0{,}33 - 1{,}67) - 1 \cdot 80] + 8{,}49(-169{,}71) \cdot 0{,}47 + 16{,}97[-0{,}47(169{,}71 - 169{,}71 - 169{,}71)] = -5083{,}60$$

b) Pomeranje čvora "8" usled temperaturne promene



-Slobodni članovi

$$EFc\delta_{1t} = EFc \sum S_1 \alpha_t t° l = 0$$

-Statički neodređena veličina

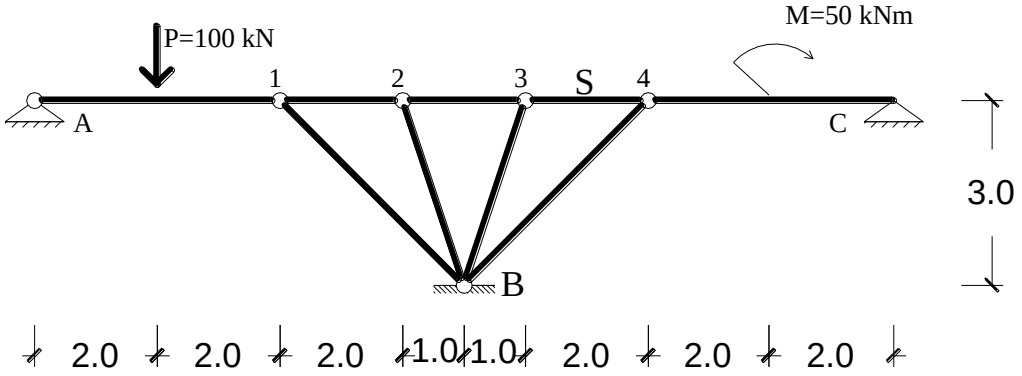
$$X_{1,t} = -\frac{\delta_{1t}}{\delta_{11}} = 0$$

-Sile u štapovima $St = S_1 X_{1,t} = 0$

$$v_8 = \sum \frac{S_t \bar{S}_v}{EF} l + \sum \bar{S}_V \alpha_t t° l = 12 \cdot 20 \cdot 10^{-5}(0{,}66 + 1{,}33 + 2) = 0{,}0096m$$

Zadatak: Za nosač na slici odrediti obrtanje štapa "S" usled jednovremenog delovanja datog opterećenja i pomeranja oslonca "C" u levo za 5cm.

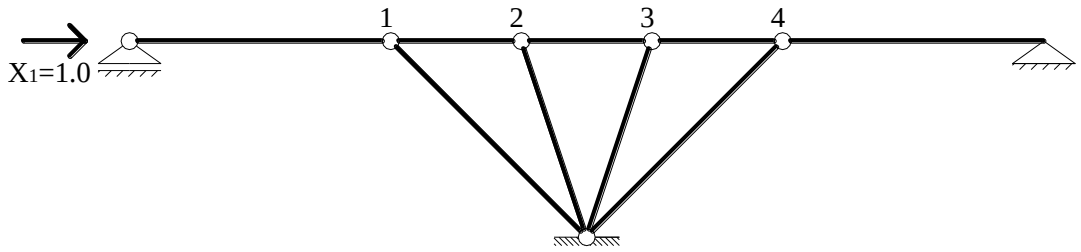
Geometrijske karakteristike: grede A-1 i 4-C: 20/40cm
ostali štapovi: 20/20cm
$E = 3 \cdot 10^7\ kN/m^2.$



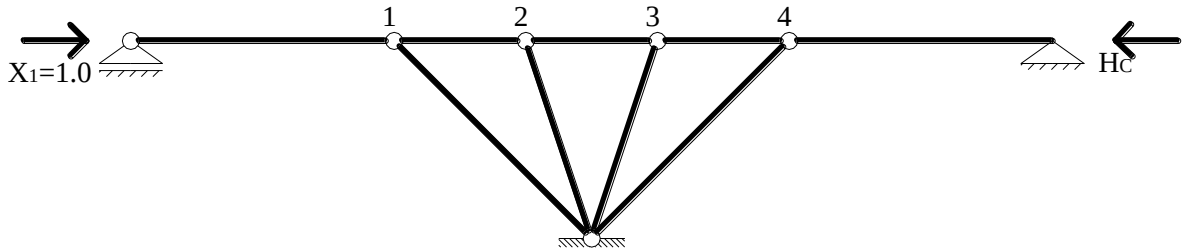
- *Statička neodređenost nosača*

$$\boldsymbol{n = Z_s + Z_k + Z_o + Z_u - 2K = 9 + 0 + 6 + 0 - 2 \cdot 7 = 1\ x\ stat.noedređen\ nosač}$$

-Izbor osnovnog sistema



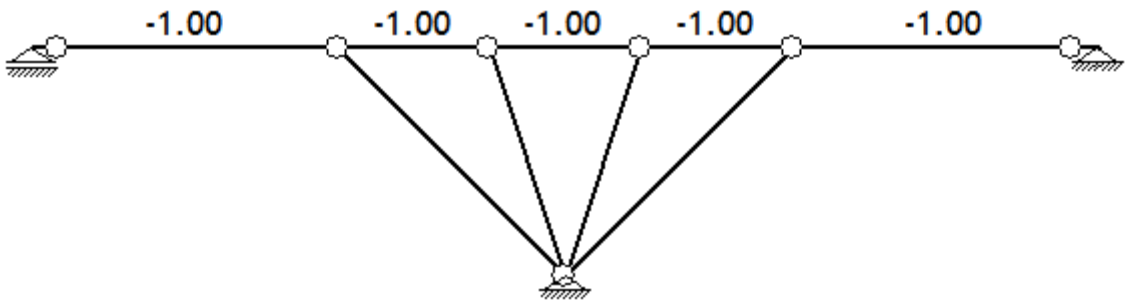
-Reakcije oslonaca usled statički nepoznate X_1=1.0



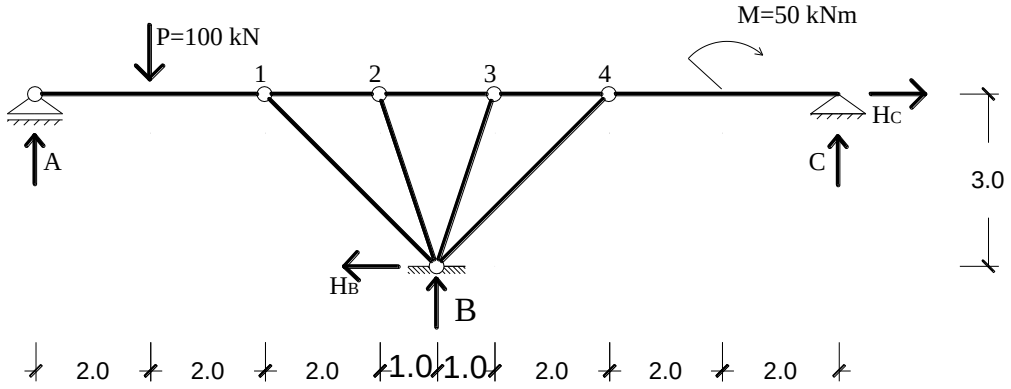
1.$\sum M_1{}^l = 0 \rightarrow A = 0$
2.$\sum M_4{}^d = 0 \rightarrow C = 0$
3.$\sum M_B = 0 \rightarrow H_C = 1.0$
4.$\sum H_i = 0 \rightarrow H_B = 0$
5.$\sum V_i = 0 \rightarrow B = 0$

Opterećenje na osnovnom sistemu ne izaziva transverzalne sile i moment savijanja.

- *Dijagram normalnih sila($\boldsymbol{N_1}$)*

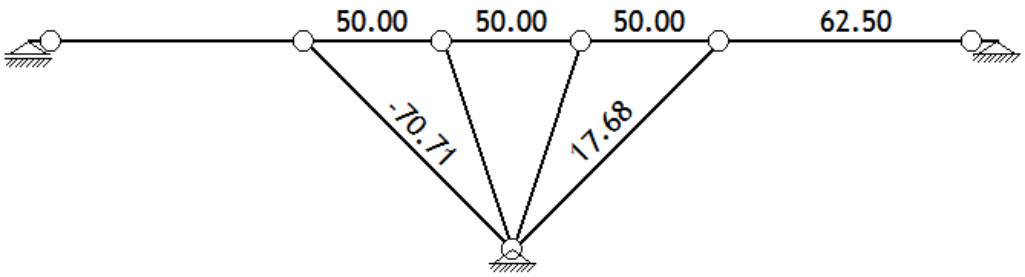


-Reakcije oslonaca na osnovnom sistemu usled zadatog opterećenja



1.$\sum M_1{}^l = 0 \rightarrow A = 50kN$
2.$\sum M_4{}^d = 0 \rightarrow C = 12.5kN$
3.$\sum M_B = 0 \rightarrow H_C = 62.5kN$
4.$\sum H_i = 0 \rightarrow H_B = 62.5kN$
5.$\sum V_i = 0 \rightarrow B = 37.5kN$

-Dijagram normalnih sila usled zadatog opterećenja ($\boldsymbol{N_o[kN]}$)



-<u>Geometrijske karakteristike</u>

$$F_c = 0.2 \cdot 0.2 = 0.04\,m^2\,, I_c = 0.2 \cdot \frac{0.2^3}{12} = 1.33 \cdot 10^{-4} m^4\,, \frac{I_c}{F_c} = 3.33 \cdot 10^{-3} m^2, \qquad EI_c = 4000 kNm^2$$

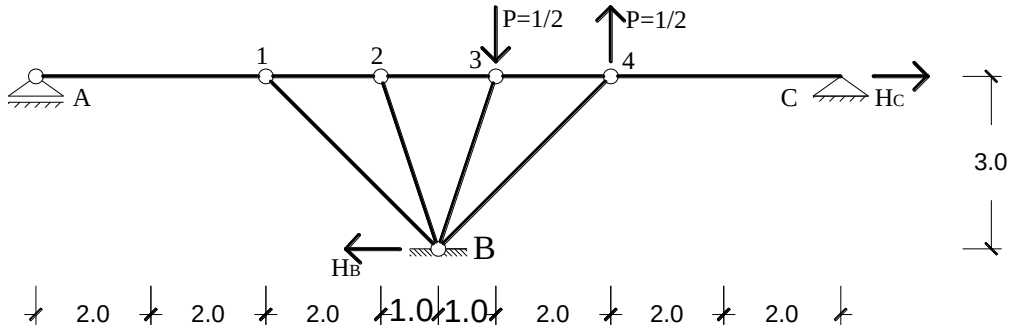
-Uslovna jednačina metode sila

$$\delta_{10} + \delta_{1C} + \delta_{11} X_1 = 0$$
$$EI_c \delta_{10} = \frac{I_c}{F_c} \int N_1 N_0 \frac{F_c}{F} d_s = 3.33 \cdot 10^{-3}[-1 \cdot 50(2+2+2) - 1 \cdot 62.5 \cdot 2] = -1.417$$
$$EI_c \delta_{1C} = -EI_c \sum \bar{C}_i c_i = -4000 \cdot 1 \cdot 0.05 = -200$$
$$EI_c \delta_{11} = \frac{I_c}{F_c} \int N_1\, N_1 \frac{F_c}{F} ds = 3.33 \cdot 10^{-3}[1^2(2+2+2+2+2)] = 0.03333$$

-Uslovna jednačina metode sila

$$\boldsymbol{\delta_{10} + \delta_{1C} + \delta_{11} X_1 = 0 \rightarrow \boxed{X_1 = \frac{201.42}{0.0333} = 6042.66\,kN}}$$

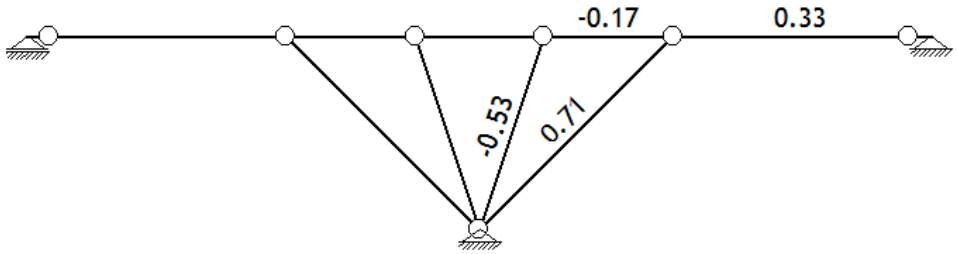
-Uticaji na nosaču usled generalisane sila za obrtanje štapa "S"



1.$\sum M_1{}^{l} = 0 \rightarrow A = 0$
2.$\sum M_4{}^{d} = 0 \rightarrow C = 0$
3.$\sum M_B = 0 \rightarrow H_C = 0.33$
4.$\sum H_i = 0 \rightarrow H_B = 0.33$
5.$\sum V_i = 0 \rightarrow B = 0$

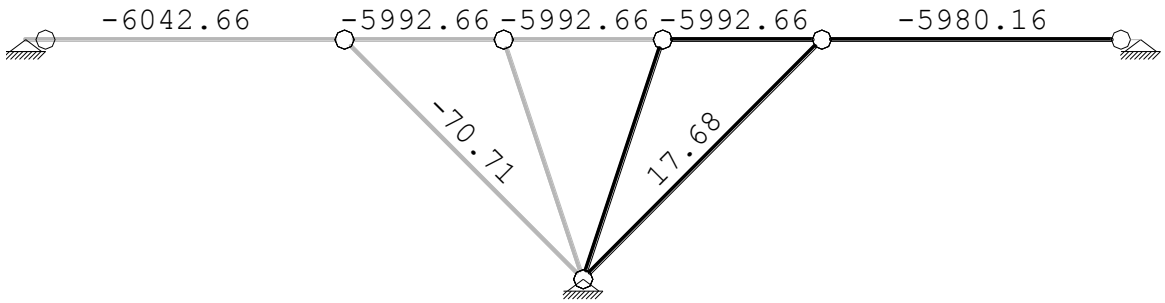
Generalisano opterećenje na osnovnom sistemu ne izaziva transverzalne sile i moment savijanja.

- *Dijagram normalnih sila usled generalisane sile* $(\overline{\boldsymbol{N}})$



- *Dijagram normalnih sila na zadatom statički neodređenom nosaču(N[kN])*

$\boldsymbol{Z = Z_0 + Z_1 X_1}$



-Obrtanje štapa S

$$\boldsymbol{EI_c\varphi = \frac{I_c}{F_c}\int N\,\overline{N}\,\frac{F_c}{F}\,d_s - EI_c\sum \overline{C}_i c_i}$$

$$\boldsymbol{= 3.33\cdot 10^{-3}[17.68\cdot 0.71\cdot 4.24 + 5992.66\cdot 0.17\cdot 2 - 5980.16\cdot 0.33\cdot 2] + 4000\cdot 0.33\cdot 0.05 = 60,35}$$