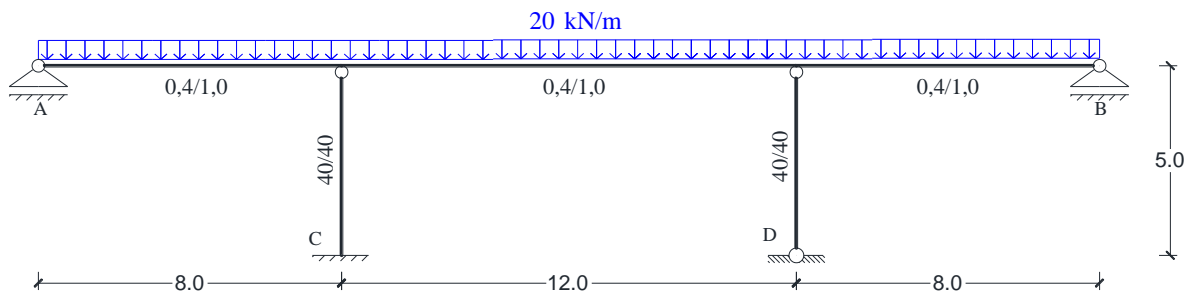


STATIKA KONSTRUKCIJA 1 - VEŽBE

Zadatak: Za nosač i opterećenje na slici odrediti dijagram presječnih sila. Uticaj normalnih sila na deformaciju zanemariti. $E = 3 \cdot 10^7 \text{ kN/m}^2$.



1. Statička neodređenost

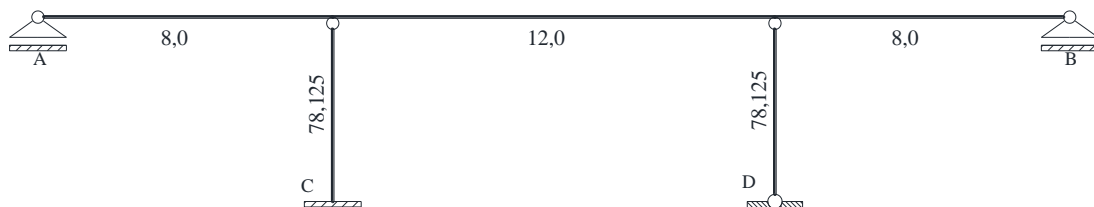
$$Z_o=6, Z_u=1, Z_k=2, Z_s=5, K=6, n = Z_s + Z_k + Z_o + Z_u - 2K = 2xstat.neod.nosač$$

2. Osnovni sistem

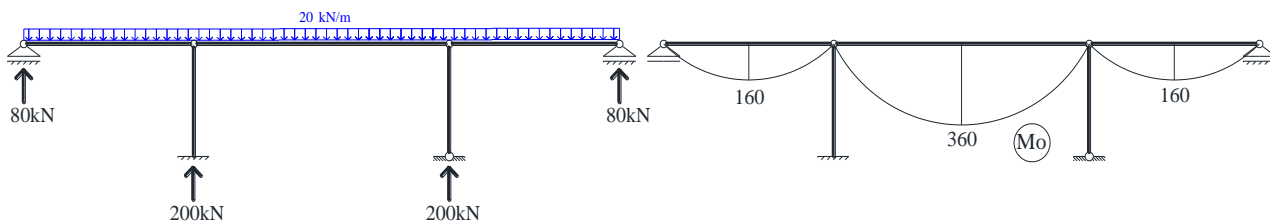


3. Redukovane dužine

$$I_c = \frac{0.4 \cdot 1.0^3}{12} = 0.0333 \text{ m}^4, \quad l' = \frac{I_c}{I} d_s$$

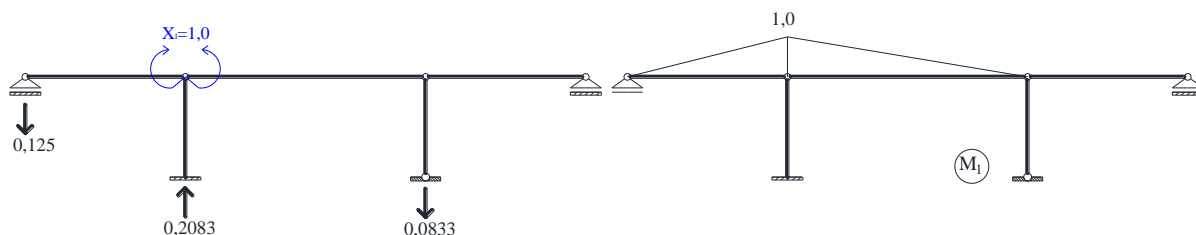


4. Uticaji usled zadatog opterećenja



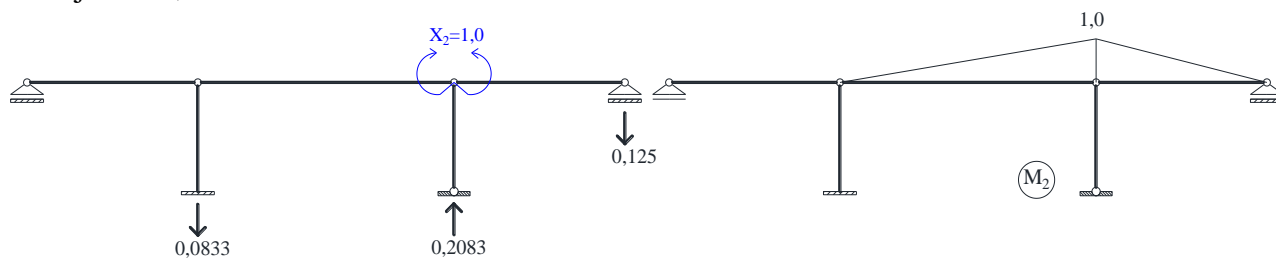
5. Uticaji od stanja $X_i=1,0$

*Stanje $X_1=1,0$



STATIKA KONSTRUKCIJA 1 - VEŽBE

*Stanje $X_2=1,0$



6. Koeficijenti uz nepoznate

$$EIc\delta_{11} = \int M_1 M_1 d'_s = \frac{1}{3} 8 \cdot 1 \cdot 1 + \frac{1}{3} 12 \cdot 1 \cdot 1 = 6.667$$

$$EIc\delta_{12} = EIc\delta_{21} = \int M_1 M_2 d'_s = \frac{1}{6} 12 \cdot 1 \cdot 1 = 2.0$$

$$EIc\delta_{22} = \int M_2 M_2 d'_s = \frac{1}{3} 12 \cdot 1 \cdot 1 + \frac{1}{3} 8 \cdot 1 \cdot 1 = 6.667$$

7. Slobodni članovi

$$EIc\delta_{10} = \int M_1 M_0 d'_s = -\frac{1}{3} 8 \cdot 1 \cdot 160 - \frac{1}{3} 12 \cdot 1 \cdot 360 = -1866.67$$

$$EIc\delta_{20} = \int M_2 M_0 d'_s = -\frac{1}{3} 12 \cdot 1 \cdot 360 - \frac{1}{3} 12 \cdot 1 \cdot 160 = -1866.67$$

8. Uslovne jednačine

$$\delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 = 0 \rightarrow 6,667X_1 + 2X_2 = 1866,67$$

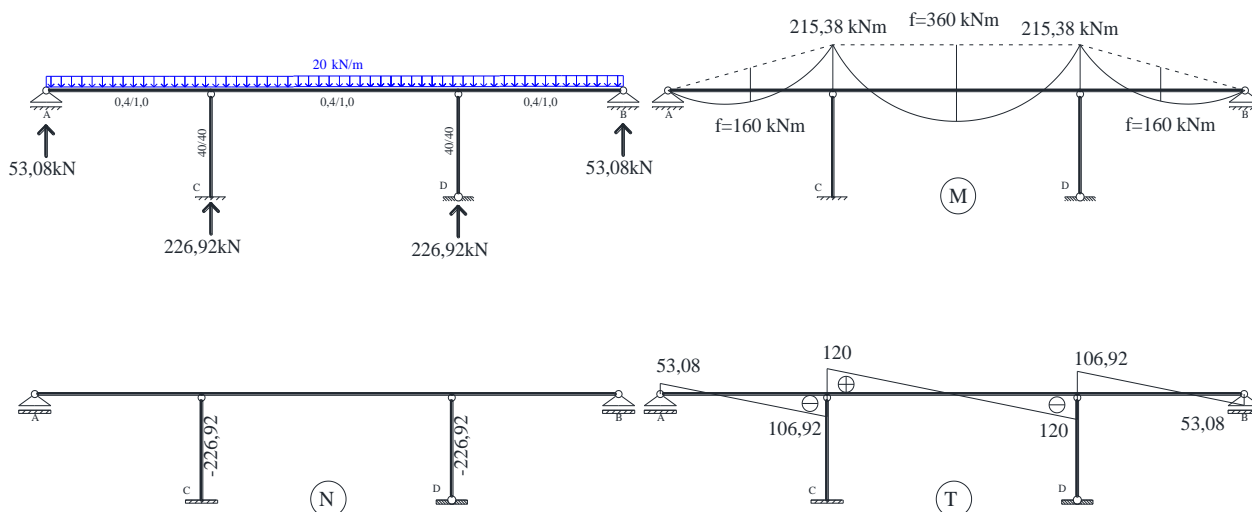
$$\delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 = 0 \rightarrow 2X_1 + 6,667X_2 = 1866,67$$



$$X_1 = X_2 = 215,38 \text{ kNm}$$

9. Dijagrami presječnih sila

$$R = R_0 + R_1X_1 + R_2X_2$$



Proračun generalisanih pomeranja

$$\delta = \int \frac{M\bar{M}}{EI} d_s + \int \frac{N\bar{N}}{EF} d_s + \int k \frac{T\bar{T}}{FG} d_s + \int \bar{M}\alpha_t \frac{\Delta t}{h} d_s + \int \bar{N}\alpha_t t^\circ d_s - \sum \bar{C}_i c_i$$

Pomjeranje usled temperature (mogu da izazovu presječne sile):

$$\delta = \int \frac{Mt\bar{M}}{EI} d_s + \int \frac{Nt\bar{N}}{EF} d_s + \int k \frac{Tt\bar{T}}{FG} d_s + \int \bar{M}\alpha_t \frac{\Delta t}{h} d_s + \int \bar{N}\alpha_t t^\circ d_s$$

Pomjeranje usled zadatih pomeranja oslonaca

$$\delta = \int \frac{Mc\bar{M}}{EI} d_s + \int \frac{Nc\bar{N}}{EF} d_s + \int k \frac{Tc\bar{T}}{FG} d_s - \sum \bar{C}_i c_i$$

Na mjestima na kojim djeluje temperaturni uticaji i gdje su N sile generalisane tada se N ne zanemaruje.

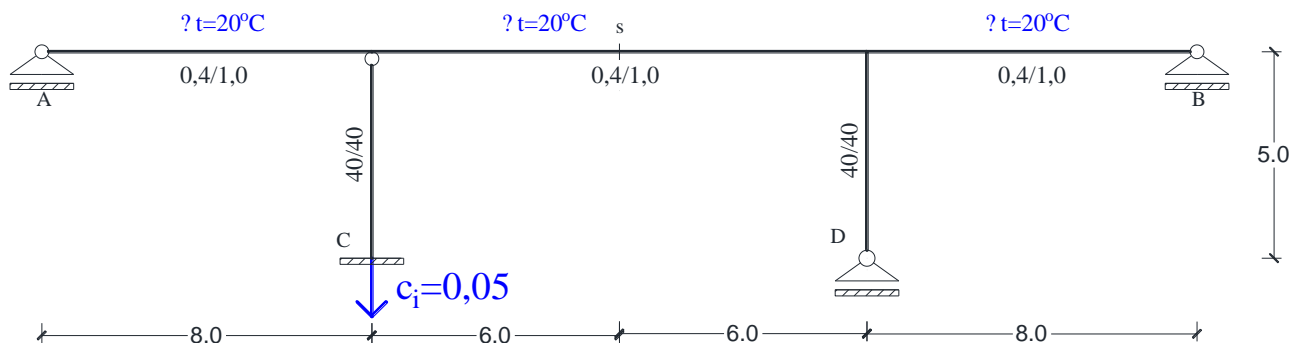
Generalisanu silu zadajemo na statički neodređenom nosaču a njen uticaj računamo na osnovnom sistemu.

Zadatak: Za nosač na skici odrediti vertikalno pomeranje za presjek „s“ usled:

a) Sleganje oslonca "C" za 5cm

b) Temperaturne promene u štapovima poteza "A-1-2-C" $\Delta t = +20^\circ\text{C}$

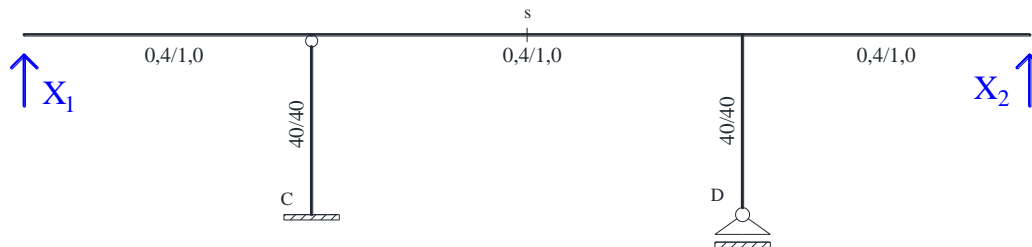
Uticaj normalnih sila na deformaciju zanemariti. $E = 3 \cdot 10^7 \text{ kN/m}^2, \alpha_t = 10^{-5} \text{ 1/}^\circ\text{C}$



1. Statička neodređenost

$$Z_o=5, Z_u=1, Z_k=3, Z_s=5, K=6, n = Z_s + Z_k + Z_o + Z_u - 2K = 2xstat.neod.nosač$$

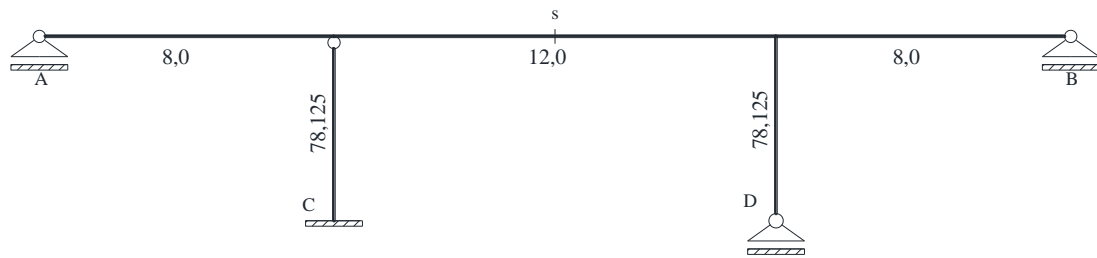
2. Osnovni sistem



3. Redukovane dužine

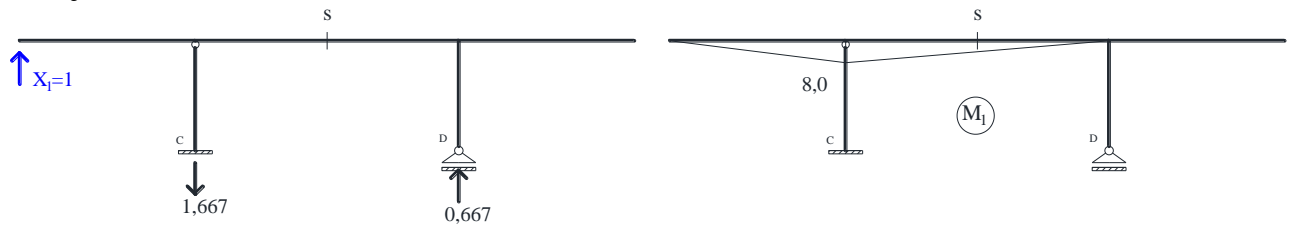
$$I_c = \frac{0.4 \cdot 1.0^3}{12} = 0.0333 \text{ m}^4, \quad l' = \frac{I_c}{I} d_s$$

STATIKA KONSTRUKCIJA 1 - VEŽBE

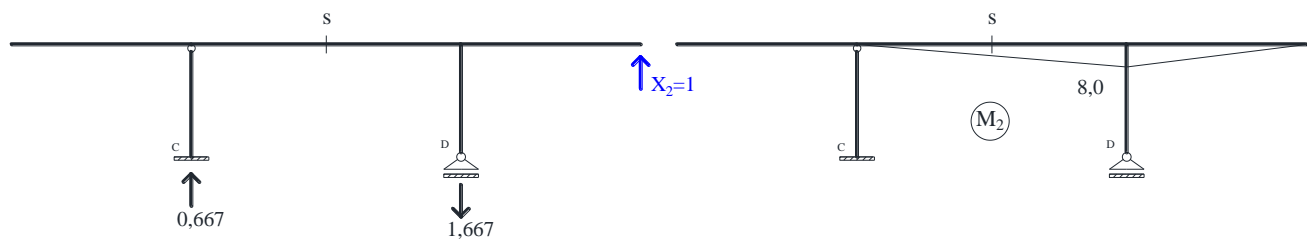


4. Uticaji od stanja $X_i=1,0$

*Stanje $X_1=1,0$



*Stanje $X_2=1,0$



5. Koeficijenti uz nepoznate

$$E I c \delta_{11} = \int M_1 M_1 d'_s = \frac{1}{3} \cdot 8 \cdot 8 \cdot 8 + \frac{1}{3} \cdot 12 \cdot 8 \cdot 8 = 426,66$$

$$E I c \delta_{12} = E I c \delta_{21} = \int M_1 M_2 d'_s = \frac{1}{6} \cdot 12 \cdot 8 \cdot 8 = 128$$

$$E I c \delta_{22} = \int M_2 M_2 d'_s = \frac{1}{3} \cdot 12 \cdot 8 \cdot 8 + \frac{1}{3} \cdot 8 \cdot 8 \cdot 8 = 426,66$$

6. Generalisana sila za vertikalno pomeranje presjeka „s“



a) Sleganje oslonca „C“

Slobodni članovi

$$E I c \delta_{1c} = -E I c \sum \bar{c}_i c_i = -10^6 \cdot (-1,66) \cdot (-0,05) = -83333,33$$

$$E I c \delta_{2c} = -E I c \sum \bar{c}_i c_i = -10^6 \cdot 0,66 \cdot (-0,05) = 33333,33$$

STATIKA KONSTRUKCIJA 1 - VEŽBE

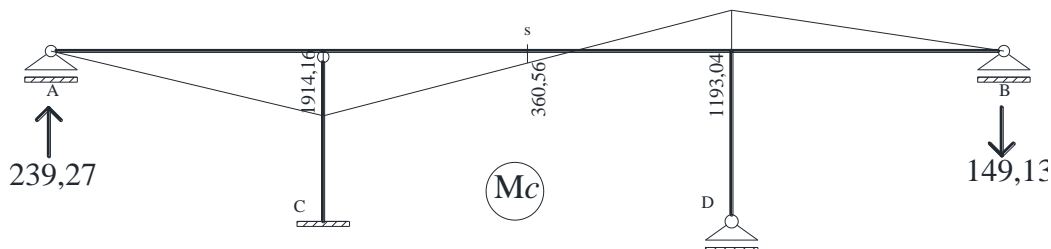
Uslovne jednačine

$$\begin{aligned} \delta_{1c} + \delta_{11}X_1 + \delta_{12}X_2 &= 0 \rightarrow 426,6X_1 + 128X_2 = 83333,3 \\ \delta_{2c} + \delta_{21}X_1 + \delta_{22}X_2 &= 0 \rightarrow 128X_1 + 426,6X_2 = -33333,3 \end{aligned}$$

$$\begin{aligned} X_1 &= 239,27 \text{ kN} \\ X_2 &= -149,13 \text{ kN} \end{aligned}$$

Dijagram momenata savijanja

$$M_c = M_1X_{1,c} + M_2X_{2,c}$$



Vetrikalno pomeranje

$$\begin{aligned} EICv &= \int Mc\bar{M}v d'_s - EIC \sum \bar{C}_i c_i \\ &= \frac{1}{2} \cdot 12 \cdot \frac{3}{2} \cdot 1914,16 + \frac{1}{2} \cdot 12 \cdot \frac{3}{2} \cdot (-1193,04) - 10^6 \cdot 0,5 \cdot (-0,05) = 31490,08 \end{aligned}$$

$$v = \frac{31490,08}{10^6} = 0,0315 \text{ m}$$

b) Temperaturne promene

Slobodni članovi

$$EIC\delta_{1t} = -EIC \int M_1\alpha_t \frac{\Delta t}{h} d_s = 10^6 \cdot \left[\frac{1}{2} \cdot 8 \cdot 8 \cdot 10^{-5} \frac{20}{1,0} + \frac{1}{2} \cdot 12 \cdot 8 \cdot 10^{-5} \frac{20}{1,0} \right] = 16000 = EIC\delta_{2t}$$

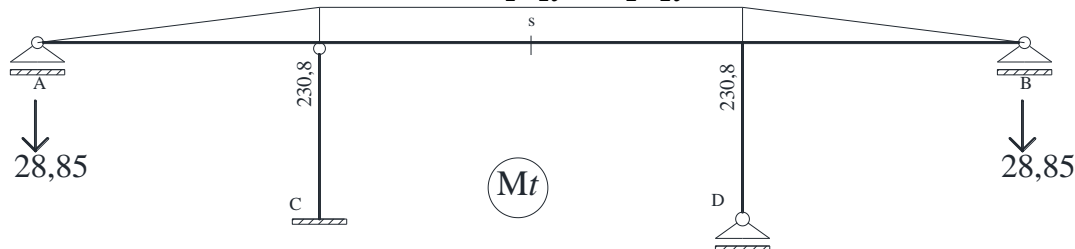
Uslovne jednačine

$$\begin{aligned} \delta_{1t} + \delta_{11}X_1 + \delta_{12}X_2 &= 0 \rightarrow 426,6X_1 + 128X_2 = -16000 \\ \delta_{2t} + \delta_{21}X_1 + \delta_{22}X_2 &= 0 \rightarrow 128X_1 + 426,6X_2 = -16000 \end{aligned}$$

$$\begin{aligned} X_1 &= -28,85 \text{ kN} \\ X_2 &= -28,85 \text{ kN} \end{aligned}$$

Dijagram momenata savijanja

$$M_t = M_1X_{1t} + M_2X_{2t}$$



Vetrikalno pomeranje

$$\begin{aligned} EICv &= \int M_t\bar{M}v d'_s + EIC \int \bar{M}v\alpha_t \frac{\Delta t}{h} d_s = -12 \cdot 230,8 \cdot 1,5 + 2 \cdot 10^6 \left(\frac{1}{2} \cdot 6 \cdot 3 \cdot 10^{-5} \frac{20}{1,0} \right) \\ &= -554,4 \end{aligned}$$

$$v = \frac{-554,4}{10^6} = -0,000554 \text{ m}$$