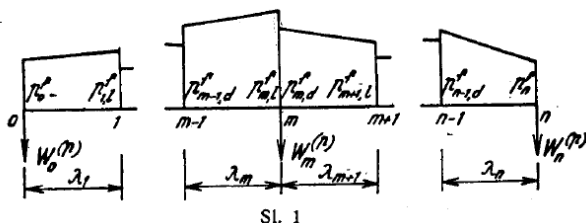


**Elastične težine**

Kada je opterećenje fiktivnog nosača komplikovano, uticaje  $T^f$  i  $M^f$  određujemo numerički. Pri tome uticaje od  $p^f$  i  $m^f$  zamenjujemo koncentrisanim silama u tačkama za koje tražimo pomeranja, odnosno obrtanja. Te sile obilježavamo sa  $W$  i nazivamo ih *elastičnim težinama*.

**1. Linearna promjena između čvorova**

- Fiktivno opterećenje  $p^f$



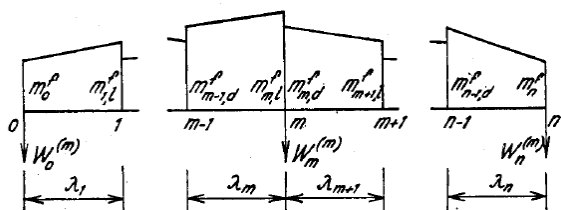
$$W_0^{(p)} = \frac{\lambda_1}{6} (2 p_0^f + p_1^f),$$

$$W_m^{(p)} = \frac{\lambda_m}{6} (p_{m-1,d}^f + 2 p_{m,l}^f) + \frac{\lambda_{m+1}}{6} (2 p_{m,d}^f + p_{m+1,l}^f),$$

$m = 1, 2, \dots, m-1$

$$W_n^{(p)} = \frac{\lambda_n}{6} (p_{n-1,d}^f + 2 p_n^f).$$

- Fiktivno opterećenje  $m^f$



$$W_0^{(m^f)} = - \frac{m_0^f + m_1^f}{2}$$

$$W_m^{(m^f)} = \frac{m_{m-1,d}^f + m_{m,l}^f}{2} - \frac{m_{m,d}^f + m_{m+1,l}^f}{2},$$

$m = 1, 2, \dots, n-1,$

$$W_n^{(m^f)} = \frac{m_{n-1,d}^f + m_n^f}{2}.$$

• Ukoliko nema skokova kod opterećenja

$$W_0^{(p)} = \frac{\lambda}{6} (2 p_0^f + p_1^f),$$

$$W_m^{(p)} = \frac{\lambda}{6} (p_{m-1}^f + 4 p_m^f + p_{m+1}^f), \quad m = 1, 2, \dots, n-1,$$

$$W_n^{(p)} = \frac{\lambda}{6} (p_{n-1}^f + 2 p_n^f),$$

$$W_0^{(m^f)} = - \frac{m_0^f + m_1^f}{2},$$

$$W_m^{(m^f)} = \frac{m_{m-1}^f - m_{m+1}^f}{2}, \quad m = 1, 2, \dots, n-1,$$

$$W_n^{(m^f)} = \frac{m_{n-1}^f + m_n^f}{2}.$$

**2. Promjena opterećenja po zakonu kvadratne parabole**

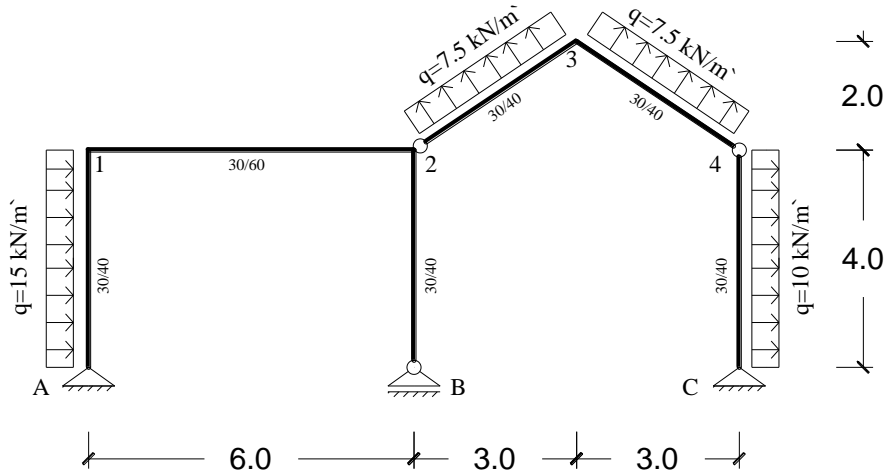
$$W_0^{(p)} = \frac{\lambda}{24} (7 p_0^f + 6 p_1^f - p_2^f),$$

$$W_m^{(p)} = \frac{\lambda}{12} (p_{m-1}^f + 10 p_m^f + p_{m+1}^f), \quad m = 1, 2, \dots, n-1,$$

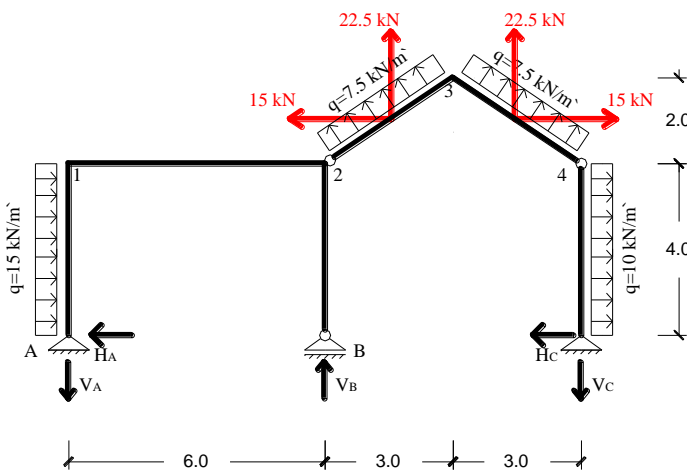
$$W_n^{(p)} = \frac{\lambda}{24} (7 p_n^f + 6 p_{n-1}^f - p_{n-2}^f).$$

# STATIKA KONSTRUKCIJA 1 - VEŽBE

**Zadatak:** Za nosač na slici odrediti dijagram vertikalnog pomeranja poteza 1-2-3-4, sa ordinatama na svakih 1.5m, usled jednovremenog delovanja datog opterećenja i temperature u osama štapova A-1, B-2 ( $t^o = +25^oC$ ). Geometrijske karakteristike poprečnih presjeka date su na slici. Uticaj normalnih sila na deformaciju zanemariti.  $E = 3 \cdot 10^7 \text{ kN/m}^2$ ,  $\alpha_t = 10^{-5} \frac{1}{^oC}$

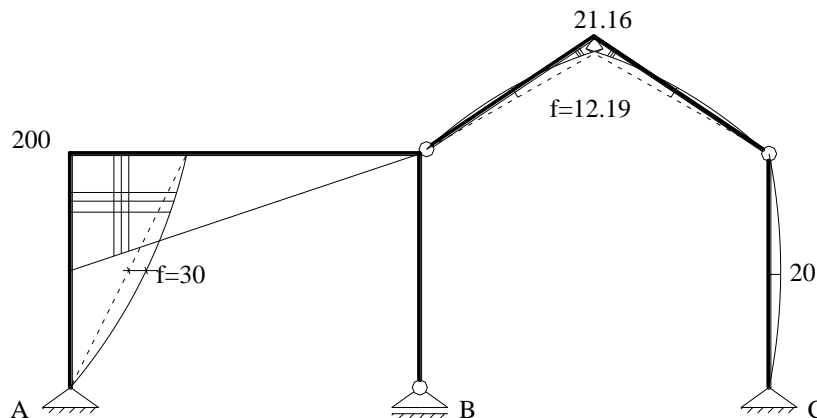


a) Reakcije oslonaca i dijagrami presečnih sila



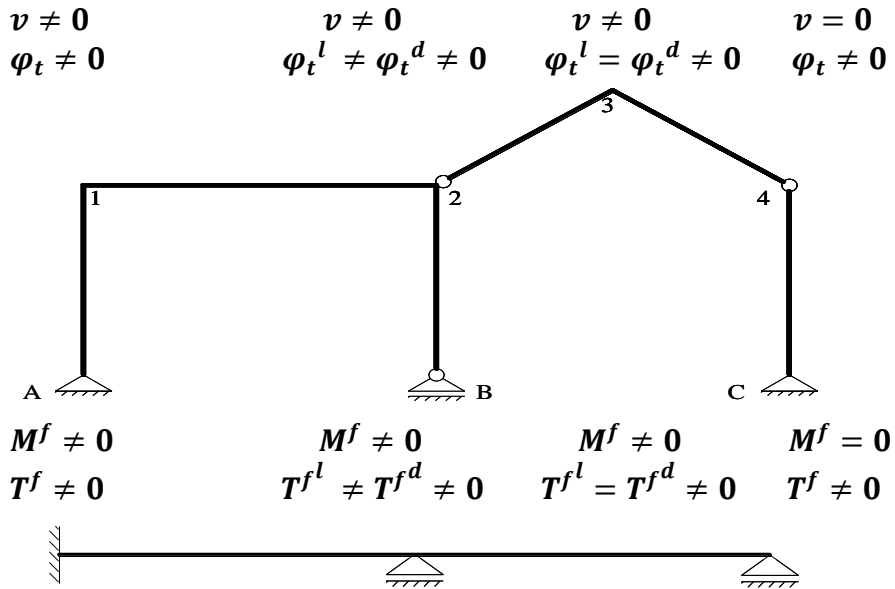
1.  $\sum M_4^{\text{dole}} = 0 \rightarrow -H_C \cdot 4 + 10 \cdot 4 \cdot 2 = 0 \rightarrow H_C = 20 \text{ kN}$
2.  $\sum M_2^{\text{desno}} = 0 \rightarrow -V_C \cdot 6 - 20 \cdot 4 + 10 \cdot 4 \cdot 2 + 22.5 \cdot (4.5 + 1.5) = 0 \rightarrow V_C = 22.5 \text{ kN}$
3.  $\sum H_i = 0 \rightarrow H_A = 80 \text{ kN}$
4.  $\sum M_2^{\text{levo}} = 0 \rightarrow -V_A \cdot 6 + 80 \cdot 4 - 15 \cdot 4 \cdot 2 \rightarrow V_A = 33.33 \text{ kN}$
5.  $\sum V_i = 0 \rightarrow V_B = 10.83 \text{ kN}$

- dijagram momenata savijanja ( $M$  [kNm])



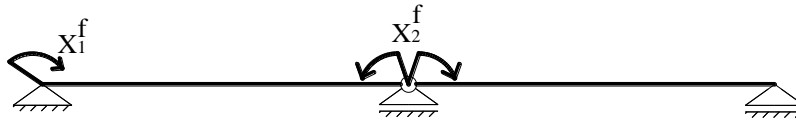
# STATIKA KONSTRUKCIJA 1 - VEŽBE

- *Određivanje fiktivnog nosača*

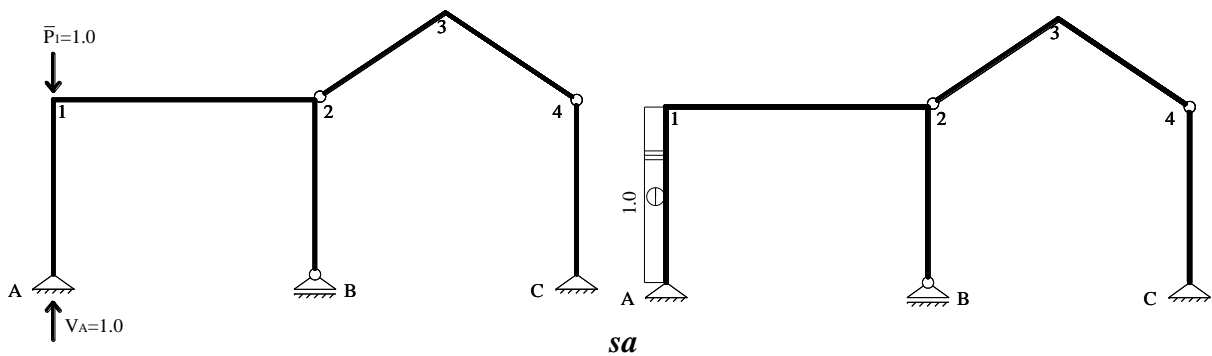


$$n = Z_s + Z_k + Z_o + Z_u - 2K = 2 + 1 + 4 + 1 - 2 \cdot 3 = 2 \text{ x stat. neodređen fiktivni nosač}$$

- *Osnovni sistem datog statički neodređenog fiktivnog nosača*



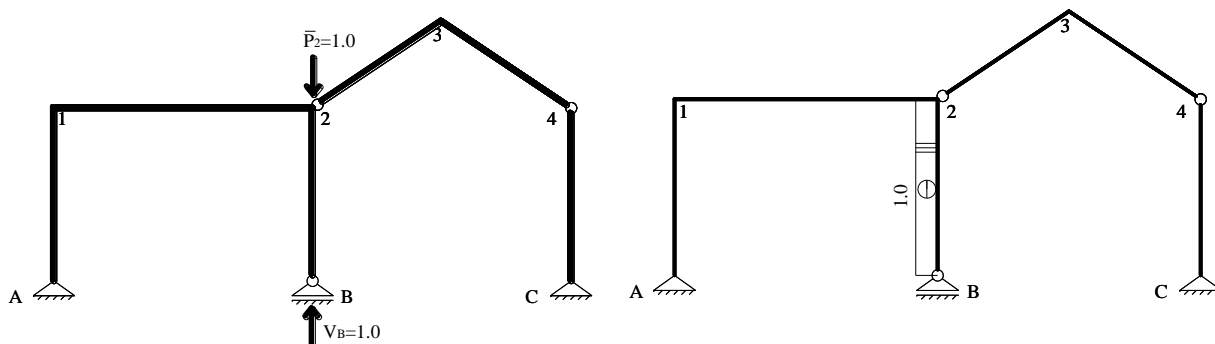
- *Statički neodređena fiktivna veličina  $X_1^f$  - Dijagram normalnih sila usled generalisane sile ( $\bar{N}$ )*



$$X_1^f = \int \bar{N} \alpha_t t^o d_s = -1 \cdot 25 \cdot 10^{-5} \cdot 4 = -1 \cdot 10^{-3}$$

# STATIKA KONSTRUKCIJA 1 - VEŽBE

- Statički neodređena fiktivna veličina  $X_2^f$  - Dijagram normalnih sila usled generalisane sile ( $\bar{N}$ )

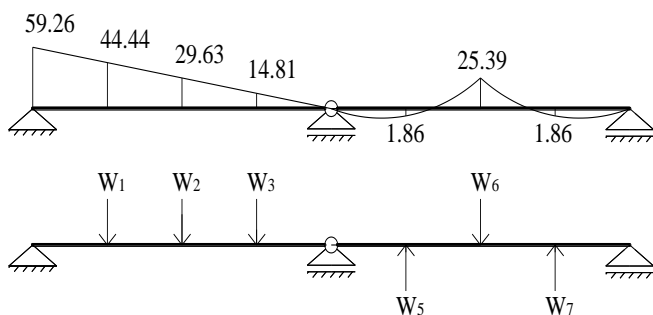


$$X_2^f = \int \bar{N} \alpha_t t^0 d_s = -1 \cdot 25 \cdot 10^{-5} \cdot 4 = -1 \cdot 10^{-3}$$

- Fiktivno opterećenje raspodeljenim silama

$$I_c = \frac{0.3 \cdot 0.4^3}{12} = 1.6 \cdot 10^{-3} m^4, \quad EI_c = 48000 kNm^2$$

$$p^f = \left( \frac{M}{EI} + \alpha_t \frac{\Delta t^0}{h} \right) \frac{1}{\cos \alpha} \rightarrow EI_c p^f = \frac{I_c}{I} M \frac{1}{\cos \alpha}$$



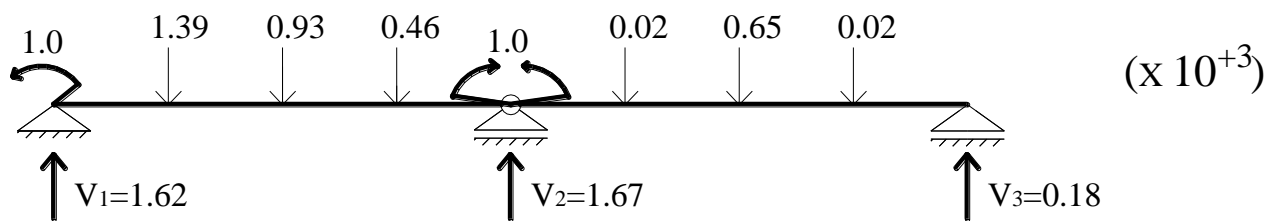
$$W_1 = \frac{1.5}{6} (59.26 + 4 \cdot 44.44 + 29.63) = 66.66$$

$$W_2 = \frac{1.5}{6} (44.44 + 4 \cdot 29.63 + 14.81) = 44.44$$

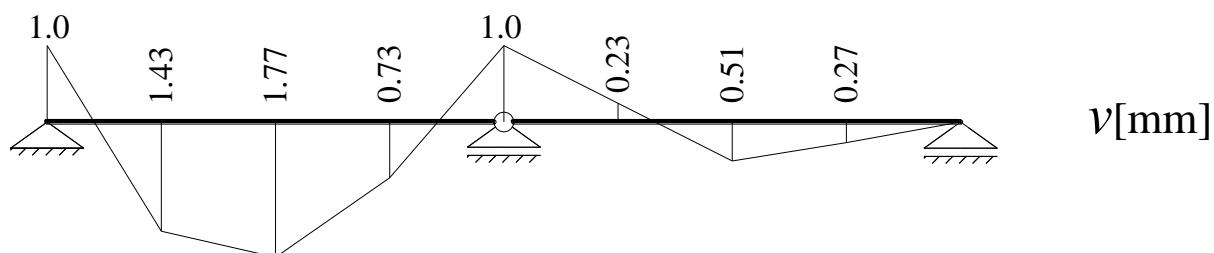
$$W_3 = \frac{1.5}{6} (29.63 + 4 \cdot 14.81 + 0) = 22.22$$

$$W_5 = W_7 = \frac{1.5}{12} (0 + 10 \cdot 1.86 - 25.39) = -0.85$$

$$W_6 = \frac{1.5}{12} (-1.86 + 10 \cdot 25.39 - 1.86) = 31.27$$



(x 10<sup>+3</sup>)



v[mm]

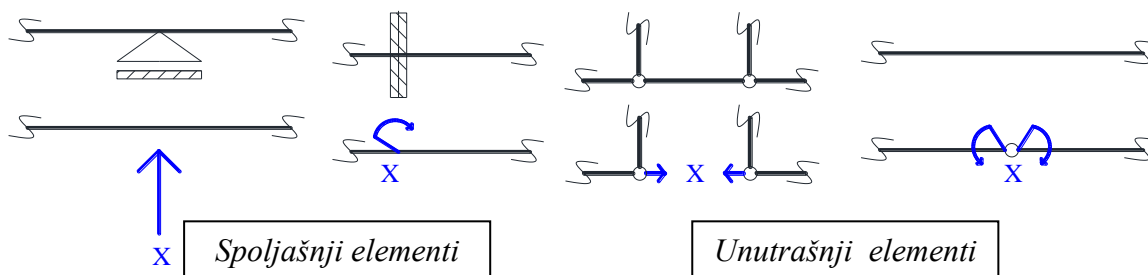
**Statički neodređeni nosači – Metod sila**

Uslovi ravnoteže: $\sum H_i = 0; \sum_{2k+m} V_i = 0; \sum M_i = 0$	Nepoznate: -Reakcije oslonaca $C_{oi} - Z_o$ -Momenti ulještenja $C_{ui} - Z_u$ -Sile u štapovima $S_{ik} - Z_s$ -Momenti $M_{ik}$ i $M_{ki} - Z_k + m$
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Kriterijum za statički neodređen nosač:  $Z_s + Z_k + Z_o + Z_u + m > 2K + m$

Statička neodređenost:  $n = Z_s + Z_k + Z_o + Z_u - 2K$

Statički neodređene veličine:  $X_1, X_2, \dots, X_n$



Jednačina statičke neodređenosti:  $\sum \delta_{ij} \cdot x_j + \delta_{i0}$

$$\delta_{ij} = \int \frac{M_i M_j}{EI} d_s + \int \frac{N_i N_j}{EF} d_s + \int k \frac{T_i T_j}{FG} d_s$$

$$\delta_{i0} = \delta_{i0} + \delta_{it} + \delta_{ic}$$

$$\delta_{i0} = \int \frac{M_i M_o}{EI} d_s + \int \frac{N_i N_o}{EF} d_s + \int k \frac{T_i T_o}{FG} d_s$$

$$\delta_{it} = \int M_i \alpha_t \frac{\Delta t}{h} d_s + \int N_i \alpha_t t^o d_s$$

$$\delta_{ic} = - \sum C_i c_i$$

$$\begin{aligned} \delta_{10} + \delta_{11}X_1 + \delta_{12}X_2 + \dots + \delta_{1n}X_n &= 0 \\ \delta_{20} + \delta_{21}X_1 + \delta_{22}X_2 + \dots + \delta_{2n}X_n &= 0 \\ \vdots & \\ \delta_{n0} + \delta_{n1}X_1 + \delta_{n2}X_2 + \dots + \delta_{nn}X_n &= 0 \end{aligned}$$

Nepoznate određujemo super pozicijom:  $Z = Z_0 + Z_1X_1 + Z_2X_2 + \dots + Z_nX_n$

Postupak proračuna:

1. Određivanje statičke neodređenosti
2. Usvajanje (izbor) osnovnog sistema
3. Određivanje redukovanih dužina
4. Dijagrami  $M_o, N_o, T_o, \alpha_t \frac{\Delta t}{h}, \alpha_t t^o, C$  od spoljašnjeg dejstva
5. Dijagrami  $M_i, N_i, T_i$  za stanja  $X_i=1,0$
6. Koeficijenti uz nepoznate  $\delta_{ij}$
7. Slobodni članovi  $\delta_{i0}$
8. Uslovne jednačine
9. Uticaji na statički neodređenom nosaču