**Deformacija nosača – primena principa virualnih sila**

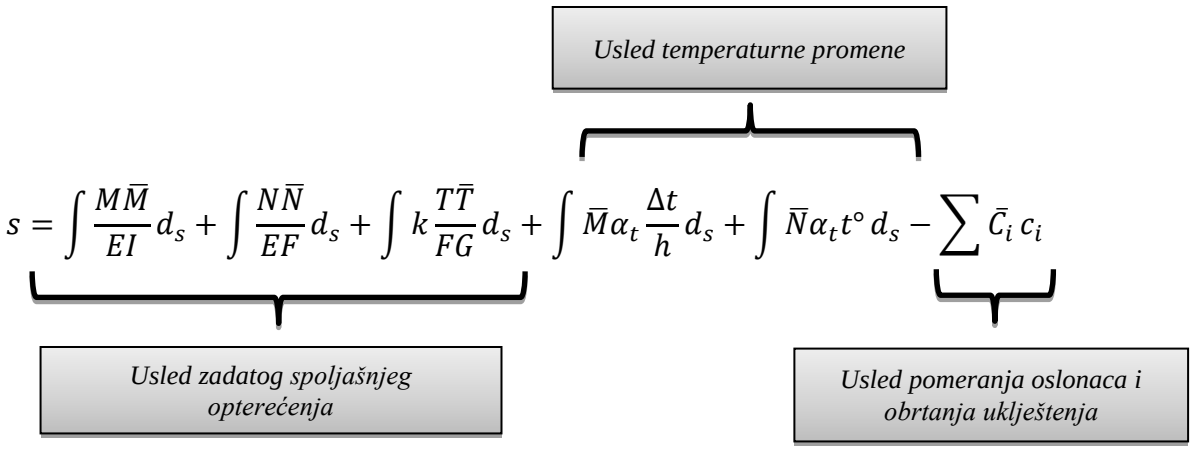
Princip virtualnih sila jednog elastičnog linijskog sistema izveden je iz veze pomeranja, obrtanja i deformacijskih veličina štapova i iz uslova kompatibilnosti:

$$\underset{\textit{rad spoljašnjih sila}}{\sum \bar{P} \cdot s + \sum \bar{C}\, c} = \underset{\textit{rad unutrašnjih sila}}{\int (\bar{M}\chi + \bar{N}\varepsilon + \bar{T}\varphi_t)\, d_s}$$

Za elastični ravan linijski nosač koji se ponaša po Hukovom zakonu, deformacijske veličine su:

$$\chi = \frac{M}{EI} + \alpha_t \frac{\Delta t}{h}$$
$$\varepsilon = \frac{N}{EF} + \alpha_t t^{\circ}$$
$$\varphi_t = k\frac{T}{FG}$$

Ukoliko na nosač djeuje jedinična generalisana sila $\bar{P} = 1$, tada je pomeranje *s*, u pravcu i smeru jedinične generalisane sile, dato izrazom:



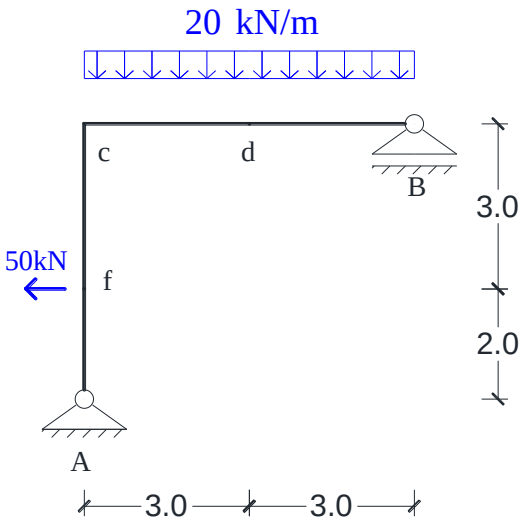

- **M,T,N** – sile u presecima usled stvarnog opterećenja
- $\bar{M}, \bar{T}, \bar{N}$ – sile u presecima usled generalisane sile
- $E$ - modul elastičnosti materijala
- $G$ - modul klizanja
- $\alpha_t$ - koeficijent temperaturne dilatacije $1/℃$
- $\Delta t$ - temperaturna razlika između donjeg i gornjeg pojasa
- $t^{\circ}$ - temperaturna promena u osi štapa
- $\bar{C}_i$ -reakcije oslonca usled jedinične generalisane sile
- $\bar{c}_i$ - zadato pomjeranje oslonca u pravcu reakcije $\bar{C}_i$

**Zadatak:** *Za nosač sa slike odrediti:*

a) *Pomjeranje čvora „f“ u pravcu oslonca „B“*
b) *Obrtanje presjeka „d“*
c) *Razmicanje tačaka „f“ i „d“*
d) *Promjenu ugla između presjeka „f“ i „d“*
e) *Promjenu ugla između štapova „f-c“ i „d-B“*

*Usled:*

1) *Zadatog opterećenja sa slike*
2) *Temperaturne promene u štapu „c-b“* ($t_u = +25°C, t_o = -5°C$)
3) *Sleganja oslonca „B“ za* $c_b = 2cm$.




- *Horizontalni štap b/h=0,3/0,5m*
- *Vertikalni štap b/h=0,3/0,4m*
- $E = 2 \cdot 10^7\ kN/m^2$
- $\alpha_t = 10^{-5}\ 1/°C$

*Generalisano pomeranje uz zanemarivanje transverzalnih sila:*

$$EI_c s = \int M\bar{M}\frac{I_c}{I}d_s + \frac{I_c}{F_c}\int N\bar{N}\frac{F_c}{F}d_s + EI_c\int \bar{M}\alpha_t\frac{\Delta t}{h}d_s + EI_c\int \bar{N}\alpha_t t°\, d_s - EI_c\sum \bar{C}_i\, c_i$$

$l'$ $\qquad$ $l''$

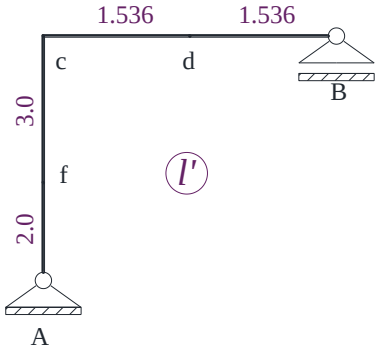
*-Geometrijske karakteristike preseka:*

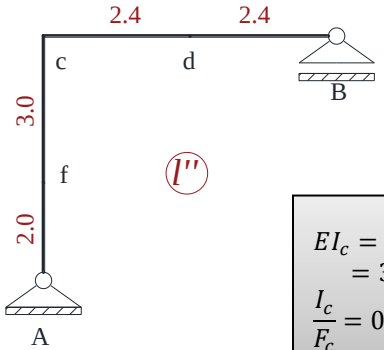
| *Štap* | *b/h [m]* | *I [$m^4$]* | *F [$m^2$]* |
|---|---|---|---|
| *a-c* | *0.3/0.4* | ***0.0016=$I_c$*** | ***0.12=$F_c$*** |
| *c-b* | *0.3/0.5* | *0.003125* | *0.15* |

*-Redukovane dužine*

$$l' = \frac{I_c}{I}d_s\ ;\quad l'' = \frac{F_c}{F}d_s$$

$I_c$*- uporedni moment inercije*
$F_c$*- uporedna površina*

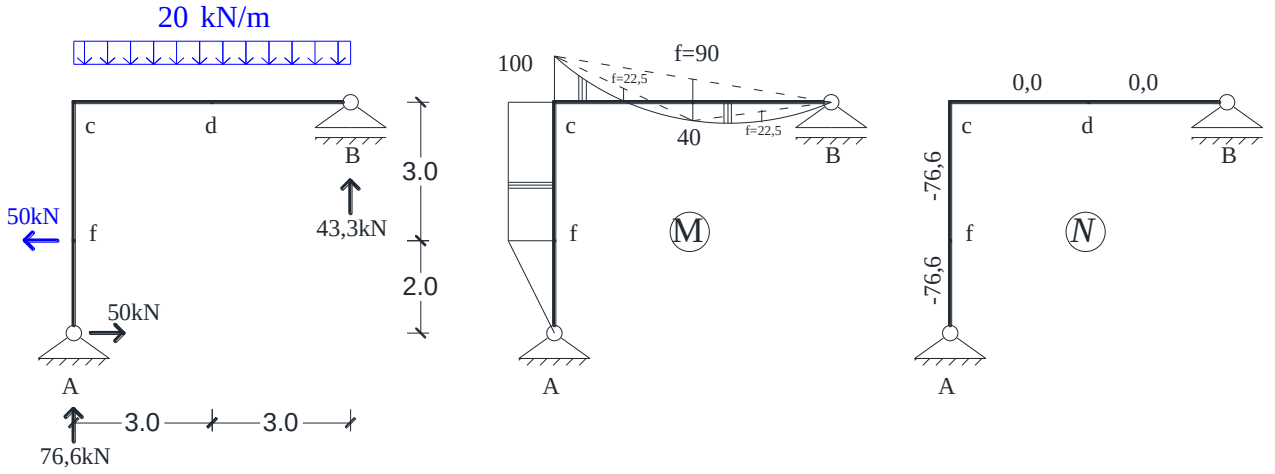


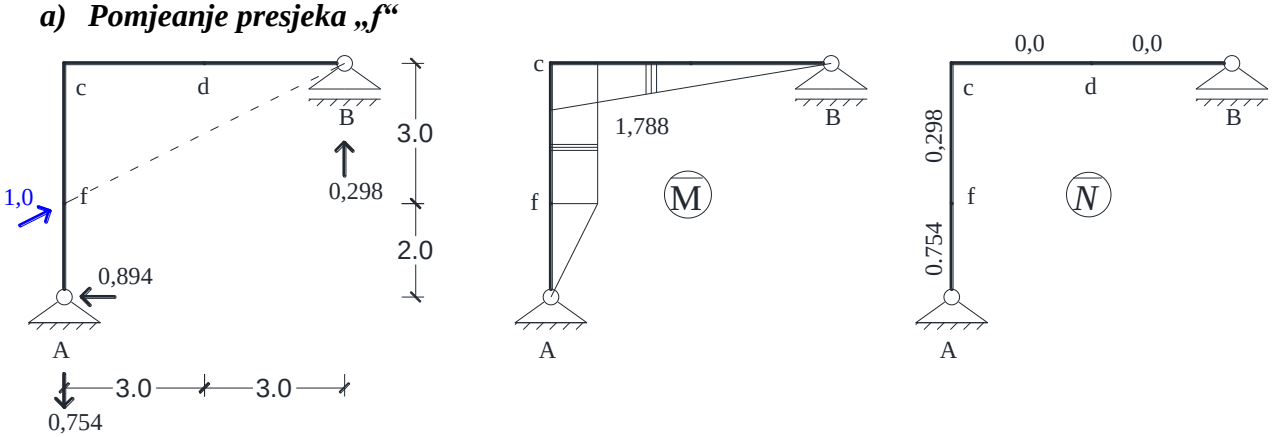

$EI_c = 2 \cdot 10^7 \cdot 0{,}016 = 32000 kNm^2$

$\frac{I_c}{F_c} = 0{,}013 m^2$

## 1. *Deformacija usled zadatog opterećenja*

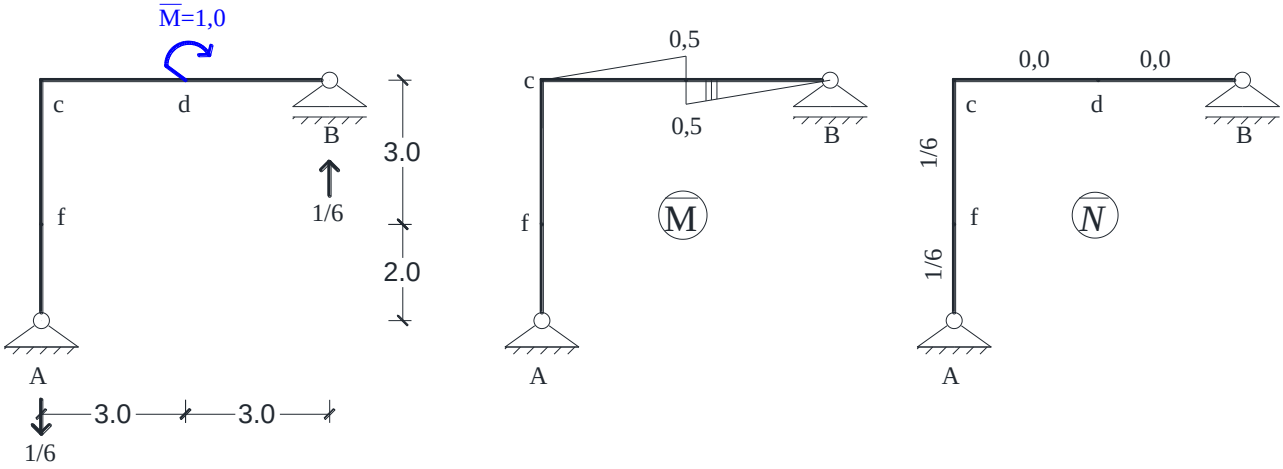



### a) *Pomjeanje presjeka „f“*



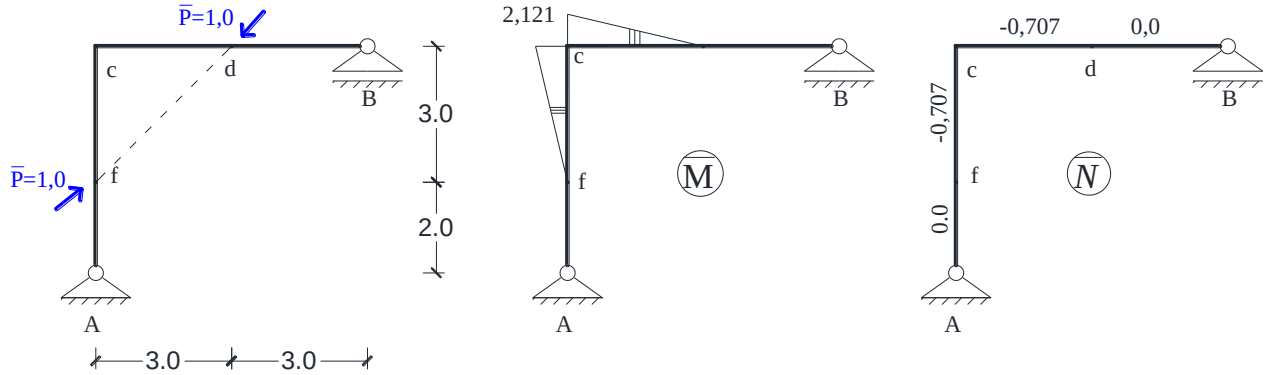

$$EI_c\delta = -\frac{1}{3}2\cdot 100\cdot 1{,}788 - 3\cdot 100\cdot 1{,}788 - \frac{1}{3}3{,}072\cdot 100\cdot 1{,}788 + \frac{1}{3}3{,}072\cdot 90\cdot 1{,}788$$
$$+ 0{,}013[2\cdot(-76{,}6)\cdot 0{,}754 + 3\cdot(-76{,}6)\cdot 0{,}298] = -676{,}35\ kNm^3$$
$$\delta = \frac{-676{,}35}{32000} = -0{,}021m$$

### b) *Obrtanje presjeka „d“*



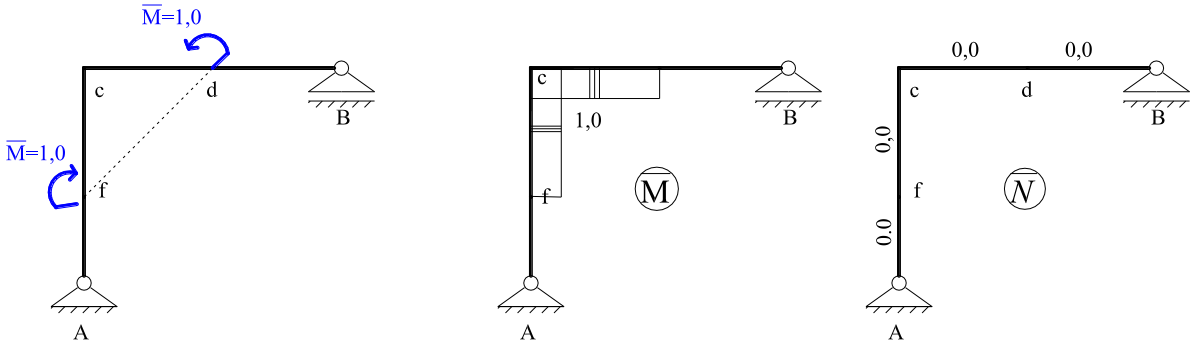

$$EI_c\varphi_d = \frac{1{,}536}{6}0{,}5\cdot(-2\cdot 40 + 100) - \frac{1{,}536}{3}\cdot 0{,}5\cdot 22{,}5 + \frac{1{,}536}{3}\cdot 0{,}5\cdot 40 + \frac{1{,}536}{3}\cdot 0{,}5\cdot 22{,}5 + 0{,}013\cdot 5$$
$$\cdot(-76{,}6)\cdot 0{,}1666 = 11{,}97 kN/m^2$$
$$\delta = \frac{11.97}{32000} = 0{,}000374\ rad$$

### c) *Razmicanje tačaka „f“ i „d“*



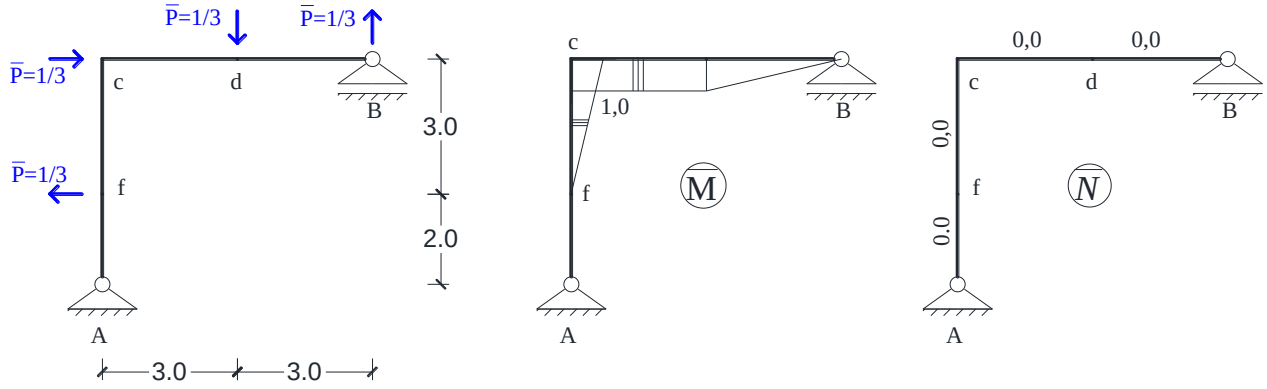

$$EI_c\Delta_{fd} = \frac{1,536}{6}2,121\cdot(2\cdot 100-40) - \frac{1,536}{3}\cdot 2,121\cdot 22,5 + \frac{3}{2}\cdot 100\cdot 2,121 + 0,013\cdot 3\cdot(-76,6)$$
$$\cdot(-0,707) = 382,71\ kNm^3$$
$$\delta = \frac{382,71}{32000} = 0,01196\ m$$

### d) *Promena ugla između „f“ i „d“*



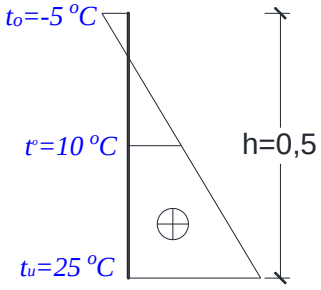

$$EI_c\Delta\varphi_{fd} = \frac{1,536}{2}1\cdot(-100+40) + \frac{2}{3}\cdot 1,536\cdot 1\cdot 22,5 - 3\cdot 100\cdot 1 = -323,04\ kN/m^2$$
$$\delta = \frac{-323,04}{32000} = -0,0101\ rad$$

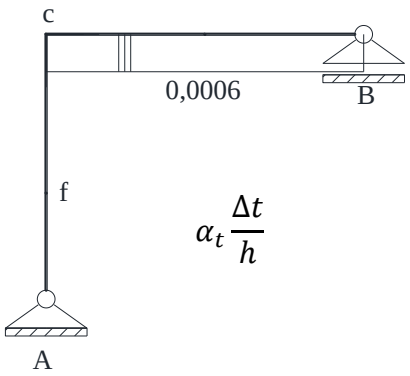
### e) *Promena ugla između štapova*



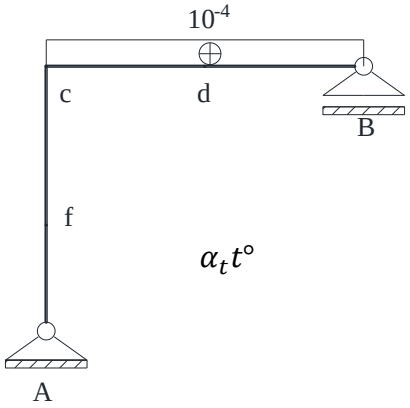

$$EI_c\Delta\varphi = -\frac{3}{2}1\cdot 100 + \frac{1,536}{2}1\cdot(-100+40) + \frac{2}{3}\cdot 1,536\cdot 22,5\cdot 1 + \frac{1,536}{2}1\cdot 40 + \frac{1,536}{2}1\cdot 22,5$$
$$= -141,04\ kN/m^2$$
$$\delta = \frac{-141,04}{32000} = -0,0044\ rad$$

## 2. *Deformacija usled temperaturne promene*




$\alpha_t \frac{\Delta t}{h}$

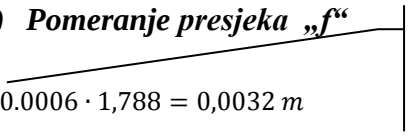



$\alpha_t t°$

$t° = 10℃$
$\Delta t = t_u - t_o = 25 + 5 = +30℃ \rightarrow$

***CRTA SE SA ONE STRANE PRESEKA GDJE JE TEMPERATURANA „+“***

### a) *Pomeranje presjeka „f“*



$\delta = \frac{6}{2} 0.0006 \cdot 1{,}788 = 0{,}0032\ m$

***Stvarna dužina!!!***

### b) *Obrtanje presjeka „d“*

$$\varphi_d = -\frac{3}{2} \cdot 0{,}5 \cdot 0.0006 + \frac{3}{2} \cdot 0{,}5 \cdot 0.0006 = 0$$

### c) *Razmicanje tačaka „f“ i „d“*

$$\Delta_{fd} = -\frac{3}{2} \cdot 2{,}121 \cdot 0.0006 + 3 \cdot (-0{,}707) \cdot 10^{-4} = -0{,}00212m$$

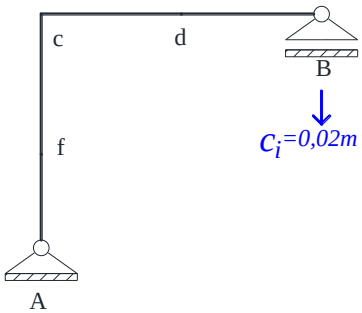
### d) *Promena ugla između „f“ i „d“*

$$\Delta\varphi_{fd} = 3 \cdot 1 \cdot 0.0006 = 0{,}0018\ rad$$

### e) *Promena ugla između štapova*

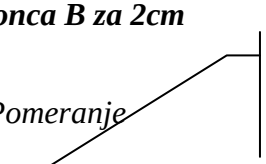
$$\Delta\varphi = 3 \cdot 1 \cdot 0.0006 + \frac{3}{2} \cdot 1 \cdot 0.0006 = 0{,}0027\ rad$$

## 3. *Deformacijia usled sleganja oslonca B za 2cm*




***$\bar{C}_i$- reakcija oslonca usled generalisane sile***
***$c_i$ – pomeranje oslonca (zadato)***

### a) *Pomeranje*



$$\delta = -\sum \bar{C}_i c_i = -[0{,}298 \cdot (-0{,}02)] = +0{,}00596m$$

### b) *Obrtanje presjeka „d“*

$$\varphi_d = -\sum \bar{C}_i c_i = -\left[\frac{1}{6} \cdot (-0{,}02)\right] = 0{,}00333\ rad$$

***c) d) e) Deformacija je nula jer ne postoji reakcija oslonca „B“ usled generalisanih sila!***