Difference Between Creep And Relaxation Poisson's Ratios: Theoretical And Practical Significance For Concrete Creep Testing

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ABSTRACT

The viscoelastic Poisson's ratio is an important parameter for assessing the multiaxial creep behavior of concrete. However, its definition in viscoelasticity can generate some ambiguity: at least five different ways of defining a viscoelastic Poisson's ratio are presented in the literature. As to their difference either in theory or in practice, little is known. In this work, we focus on the most intuitive two ways of defining a viscoelastic Poisson's ratio, which we call "relaxation Poisson's ratio" and "creep Poisson's ratio". First, we derive the analytical expressions of the two Poisson's ratios and a relationship between them. We show that their initial values are identical, and that their asymptotic values when time tends towards infinity as well. We also show that such is the case for their derivatives with respect to time. Then, considering concrete as a non-aging linear viscoelastic material, the results of multiaxial basic creep tests on concrete available in the literature are analyzed to compare the relaxation and the creep Poisson's ratios. The results show that the difference between the two Poisson's ratios is rather small but does exist in some cases. In such cases, whether this difference is significant should be considered with respect to the application considered.

INTRODUCTION

The delayed behavior of nuclear containment is an important question when the service life of these structures is discussed. The containment vessel is a biaxially prestressed structure. In this case, the viscoelastic Poisson's ratio is an important parameter for assessing the multiaxial creep behavior of concrete.

However, the definition of the time-dependent Poisson's ratio in linear viscoelasticity can generate some ambiguity: at least five different ways of defining a time-dependent viscoelastic Poisson's ratio are presented in the literature (Hilton, 2001). In this work, we focus on two ways of defining a viscoelastic Poisson's ratio that are based directly on the ratio of lateral strain $\varepsilon_l(t)$ over axial strain $\varepsilon_a(t)$. The first one, which we call relaxation Poisson's ratio ν_r, can be measured directly in the uniaxial relaxation test where the axial strain $\varepsilon_a(t)$ is kept constant, i.e., $\varepsilon_a(t) = \varepsilon_{a0}$:

$$\nu_r(t) = \frac{\varepsilon_l(t)}{\varepsilon_{a0}} \tag{1}$$

The second one, which we call creep Poisson's ratio ν_c, can be measured directly in the uniaxial creep test where the axial stress $\sigma_a(t)$ is kept constant, i.e., $\sigma_a(t) = \sigma_{a0}$:

$$\nu_c(t) = \frac{\varepsilon_l(t)}{\varepsilon_a(t)} \tag{2}$$

Hilton (2001), Tschoegl et al. (2002) and Lakes & Wineman (2006) showed that the two Poisson's ratios are not equal. How much they differ from each other has not been studied yet.

The main objective of the present study is the difference between the two Poisson's ratios. First, we derive the analytical expressions of the two Poisson's ratios as well as a relation between them. Then, we compare their initial and long-time asymptotic values. In the end, their difference during their evolution with respect to time is studied for cementitious materials by analyzing multiaxial creep test results available in literature.

POISSON'S RATIOS IN THEORY

Theoretical derivation

We restrict ourselves to non-aging linear isotropic viscoelastic materials. The general constitutive relation through which the stress tensor $\underline{\underline{\sigma}}$ (decomposed into the volumetric stress $\sigma_v = \mathrm{tr}(\underline{\underline{\sigma}})/3$ and the deviatoric stress tensor $\underline{\underline{s}} = \underline{\underline{\sigma}} - \sigma_v \underline{\underline{1}}$) is linked to the strain tensor $\underline{\underline{\varepsilon}}$ (decomposed into the volumetric strain $\varepsilon_v = \mathrm{tr}(\underline{\underline{\varepsilon}})$ and the deviatoric strain tensor $\underline{\underline{e}} = \underline{\underline{\varepsilon}} - (\varepsilon_v/3)\underline{\underline{1}}$) reads (Christensen, 1982):

$$\sigma_v(t) = K(t) \otimes \dot{\varepsilon}_v(t) \tag{3a}$$
$$s_{ij}(t) = 2G(t) \otimes \dot{e}_{ij}(t) \tag{3b}$$

where $\otimes$ holds for the convolution product defined as $f \otimes g = \int_{-\infty}^{t} f(t-\tau)g(\tau)d\tau$ and $\dot{f}$ holds for derivative with respect to time such as $\dot{f} = df(t)/dt$. Those state equations can equivalently be written (Christensen, 1982):

$$\varepsilon_v(t) = J_K(t) \otimes \dot{\sigma}_v(t) \tag{4a}$$

$$e_{ij}(t) = \frac{1}{2} J_G(t) \otimes \dot{s}_{ij}(t) \tag{4b}$$

where $J_K(t)$ and $J_G(t)$ are the bulk creep compliance and the shear creep compliance, respectively. Relaxation moduli and creep compliances are linked through $\widehat{K}\widehat{J_K} = \widehat{G}\widehat{J_G} = 1/s^2$ (Christensen, 1982), where s is the Laplace variable and $\widehat{f}(s)$ represents the Laplace transform of the function $f(t)$.

By combining Eq. 1 with Eq. 3 and solving them in Laplace domain for a uniaxial relaxation test, the relaxation Poisson's ratio ν_r is found:

$$\widehat{\nu_r}(s) = \frac{3\widehat{K}(s) - 2\widehat{G}(s)}{2s(3\widehat{K}(s) + \widehat{G}(s))} \tag{5}$$

Similarly, combining Eq. 2 with Eq. 4 and solving them directly in time domain for a uniaxial creep test, the creep Poisson's ratio ν_c is found:

$$\nu_c(t) = \frac{3J_G(t) - 2J_K(t)}{2(3J_G(t) + J_K(t))} \tag{6}$$

The elastic-viscoelastic correspondence principle (Christensen, 1982) states that by replacing the elastic parameters in an elastic relation by the s-multiplied Laplace transform of the corresponding viscoelastic parameters, one can obtain the relation between viscoelastic parameters. Applying this principle to the relation between elastic Poisson's ratio ν_0 and elastic moduli K_0 and G_0 yields Eq. 5, not Eq. 6. This means the corresponding viscoelastic parameter of elastic Poisson's ratio is the relaxation Poisson's ratio ν_r, not the creep Poisson's ratio ν_c, i.e., the elastic-viscoelastic correspondence principle can be applied only to the relaxation Poisson's ratio ν_r.

We introduce the uniaxial creep compliance $J_E(t)$ defined such that, in any uniaxial test, $\varepsilon_a(t) = J_E(t) \otimes \dot{\sigma}_a(t)$. For a uniaxial creep test, evaluating the ratio of the Laplace transform $\widehat{\varepsilon}_l(s)$ of the lateral strain over the Laplace transform $\widehat{\varepsilon}_a(s)$ of the axial strain and comparing with Eq. 2 gives:

$$\nu_c(t)J_E(t) = \nu_r(t) \otimes \dot{J}_E(t) \tag{7}$$

This formula, although derived by considering the specific case of a uniaxial creep test, is in fact generic.

Comparison of the two Poisson's ratios at initial and large times

This section is devoted to compare the two Poisson's ratios at initial time and large times. At initial time $t = 0$, the initial values of the relaxation moduli and creep compliances are equal to their elastic values: $K(t=0)=K_0$, $G(t=0)=G_0$, $J_K(t=0)=J_{K0}=K_0^{-1}$, $J_G(t=0)=J_{G0}=G_0^{-1}$. By using the initial value theorem (Auliac et al., 2000) on Eq. 5, and comparing the result with the value of Eq. 6 at $t=0$, one finds that the two Poisson's ratios are equal to the elastic Poisson's ratio ν_0:

$$\nu_r(0)=\nu_c(0)=\frac{3K_0-2G_0}{6K_0+2G_0}=\frac{3J_{G0}-2J_{K0}}{6J_{G0}+2J_{K0}}=\nu_0 \tag{8}$$

At very large times, i.e., $t\to\infty$, the bulk and shear relaxation moduli tend toward K_∞ and G_∞, respectively. Then, by using the final value theorem, the asymptotic values of creep compliances can be deduced: $J_K(t\to\infty)=1/K_\infty$, $J_G(t\to\infty)=1/G_\infty$. By using the final value theorem on Eq. 5, and comparing the result with the limit value of Eq. 6 at $t\to\infty$, one finds that the two Poisson's ratios are equal to a same asymptotic value ν_∞:

$$\nu_r(\infty)=\nu_c(\infty)=\frac{3K_\infty-2G_\infty}{6K_\infty+2G_\infty}=\nu_\infty \tag{9}$$

As to the derivative with respect to time, simplifying the heredity integral on the right side of Eq. 7, then deriving Eq. 7 with respect to time and evaluating the result at $t=0$, one finds their derivatives with respect to time are equal $\dot{\nu}_r(0)=\dot{\nu}_c(0)$. At large times, as the Poisson's ratios tend toward a finite value ν_∞, their derivatives with respect to time tend towards 0, i.e., $\dot{\nu}_r(\infty)=\dot{\nu}_c(\infty)=0$.

In conclusion, both at initial time $t=0$ and at very large times $t\to\infty$, the two Poisson's ratios are equal to each other, respectively. So are their derivatives with respect to time.

POISSON'S RATIOS IN CREEP TESTS ON CEMENTITIOUS MATERIALS

This section is devoted to compare the two Poisson's ratios from experimental creep test results that are available in literature (Bernard et al., 2003; Jordaan & Illson, 1969; Parrott, 1974). Only the "basic" creep is considered, which is the difference between the strain that takes place under autogenous condition under load and the autogenous shrinkage (Neville, 1995).

Under the assumption of linear viscoelasticity, the stress-strain relation for a multiaxial creep test can be written using either the relaxation Poisson's ratio $\nu_r(t)$ or the creep Poisson's ratio $\nu_c(t)$:

$$\varepsilon_i(t)=J_E(t)\sigma_{i0}-(\sigma_{j0}+\sigma_{k0})\nu_r(t)\otimes\dot{J}_E(t),\ \text{where } i\neq j\neq k\in\{1,2,3\} \tag{10a}$$
$$\varepsilon_i(t)=J_E(t)\sigma_{i0}-(\sigma_{j0}+\sigma_{k0})\nu_c(t)J_E(t),\ \text{where } i\neq j\neq k\in\{1,2,3\}. \tag{10b}$$

where σ_i and ε_i are the principal (constant) stresses and principal (time-dependent) strains, respectively, with $i=1,2,3$;

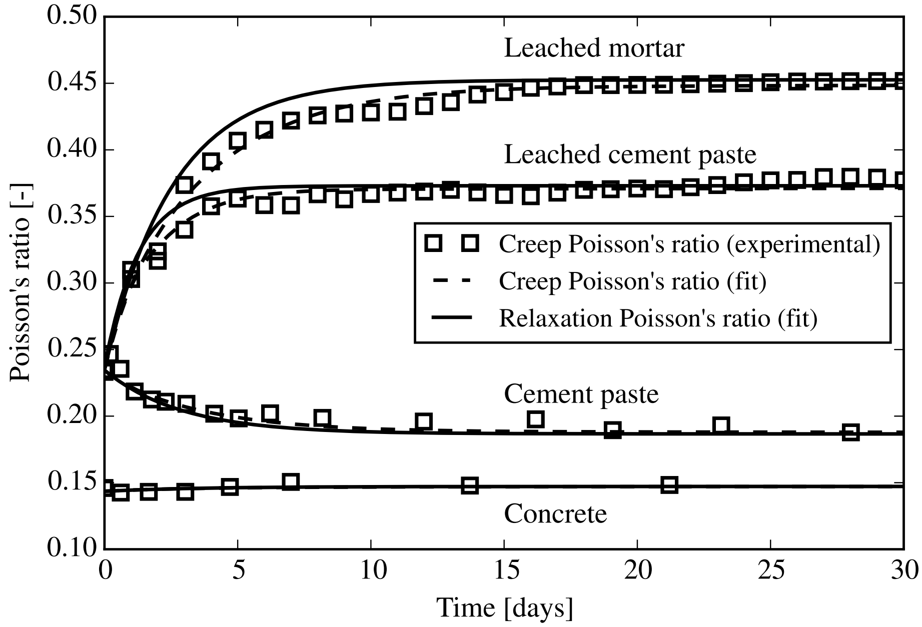

Figure 1. Experimental data of multiaxial creep tests on cementitious materials (data from (Bernard et al., 2003; Jordaan & Illson, 1969; Parrott, 1974))

One observes by comparing Eq. 10a and Eq. 10b that the creep Poisson's ratio ν_c is easier to compute from experimental results, as it does not require the calculation of a convolution integral. This explains why the creep Poisson's ratio ν_c is used more widely in the back analysis of creep experimental data than the relaxation Poisson's ratio ν_r (Jordaan & Illson, 1969, Benboudjema, F. 2002; Torrenti, J. M. et al., 2014; Hilaire, A. 2014). Using Eq. 10b, we compute experimental values of the creep Poisson's ratio ν_c and of the uniaxial creep compliance J_E. Fitting an analytical expression to the uniaxial creep compliance J_E and combining it with Eq. 7, we obtain the Poisson's ratios, which are compared to each other.

Figure 1 displays experimental data on concrete, cement paste and leached mortar and cement paste. The Poisson's ratio shows different trends for different tests. We focus on the difference between the two Poisson's ratios. The test on concrete is a biaxial creep test on a cubic sample (Jordaan & Illson, 1969): The two Poisson's ratios are almost equal during all times. The test on cement paste is a uniaxial creep test on a cuboid cement paste (Parrott, 1974): The maximum difference between the two Poisson's ratios is 0.004. The tests on leached cement paste and mortar are triaxial tests on cylindrical samples (Bernard et al., 2003): the difference between the two Poisson's ratios reaches 0.017 and 0.025 for leached cement paste and mortar, respectively. From these curves, it is observed that the difference between the two Poisson's ratios is very small when the Poisson's ratios vary little over time. On the contrary, when the variation over time is large, the difference between the two Poisson's ratios does exist but remains still rather small.

CONCLUSIONS

Two Poisson's ratios are defined for non-aging linear isotropic viscoelastic materials. Several conclusions are drawn on their difference:

- The two Poisson's ratios are material properties independent of the loading mode. They are linked to each other through Eq. 7.
- The relaxation Poisson's ratio ν_r is more convenient for solving analytically a viscoelastic problem by means of the elastic-viscoelastic correspondence principle.
- The creep Poisson's ratio ν_c is more favored in the back analysis of creep experimental data, because it can be calculated easily.
- The initial values of the two Poisson's ratios are equal, and so are their long-time asymptotic values. Similarly, the initial values of derivative with respect to time of the two Poisson's ratios are equal, and so are their long-time asymptotic values.
- Multiaxial creep test results show that the difference between the two Poisson's ratios is not significant if the Poisson's ratio varies little over time. In contrast, when variations over time are significant, a difference does exist but remains still rather small.

REFERENCES

Auliac, G., Avignant, J., Azoulay, É. (2000) *Techniques mathématiques pour la physique*, Ellipses, Paris.

Benboudjema, F. (2002) *Modélisation des déformations différées du béton sous sollicitations biaxiales. Application aux enceintes de confinement de bâtiments réacteurs des centrales nucléaires*, PhD thesis, Université de Marne-la-Vallée.

Bernard, O., Ulm, F.-J., Germaine, J. T. (2003) "Volume and deviator creep of calcium-leached cement-based materials." *Cement and Concrete Research*, 33(8):1127–1136.

Christensen, R. (1982) *Theory of viscoelasticity: an introduction*, Elsevier, Mineola, New York.

Hilaire, A. (2014) *Étude des déformations différées des bétons en compression et en traction, du jeune âge au long terme*, PhD thesis, École Normale Supérieure de Cachan.

Hilton, H. H. (2001) "Implications and constraints of time-independent Poisson ratios in linear isotropic and anisotropic viscoelasticity." *Journal of Elasticity*, 63:221–251.

Jordaan, I. J., Illson, J. M. (1969) "The creep of sealed concrete under multiaxial compressive stresses." *Magazine of Concrete Research,* 21(69):195–204.

Lakes, R. S., Wineman, A. (2006) "On Poisson's ratio in linearly viscoelastic solids." *Journal of Elasticity,* 85(1):45–63.

Neville, A. M. (1995) *Properties of concrete,* Pearson Education Limited, Essex, England.

Parrott, L. J. (1974) "Lateral strain in hardened cement paste under short- and long-term loading." *Magazine of Concrete Research*, 26(89):198–202.

Tschoegl, N. W., Knauss, W. G., Emri, I. (2002) "Poisson's ratio in linear viscoelasticity - a critical review." *Mechanics of Time-Dependent Materials*, 7(6):3–51.

Torrenti, J.-M., Benboudjema, F., Barré, F., Gallitre, E. (2014) "On the very long term delayed behaviour of concrete," *Proceedings of the International Conference on Ageing of Materials & Structures,* 218885.